Number

PreK HLC

Understanding of number values and sequences to 10 *(counting, cardinality, conservation, and stable order)*
1:1 Correspondence

September → **PreK Learning Progressions** → June

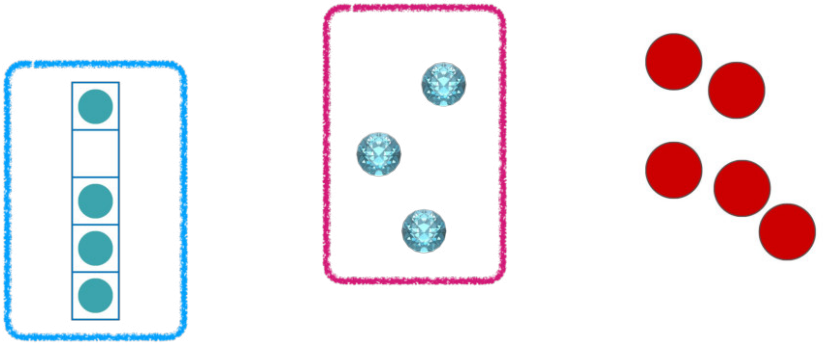
Students must use models to build understanding of the HLC and interact with a variety of contexts.

Rote Oral Count Sequence *(rote counting from 1; rote counting from any start number)*

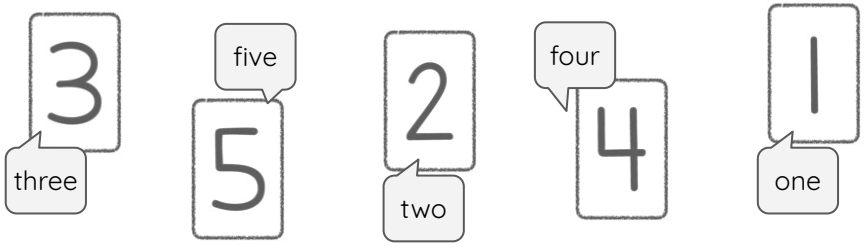
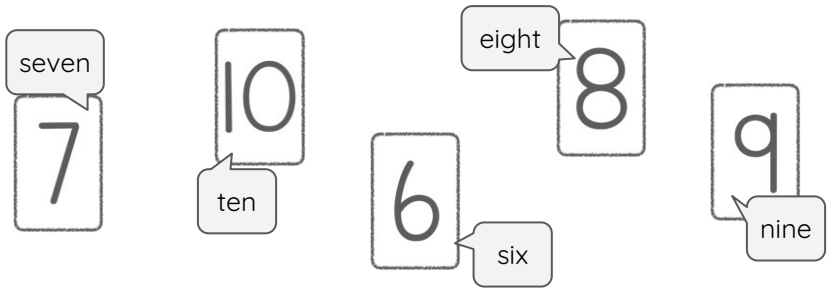
Counts Forward (FWD) from 1 to 5	Counts FWD 1 to 10
one, two, three, four, five	one, two, three, four, five, six, seven, eight, nine, ten

Counts Backward (BWD) from 3	Counts BWD from 5	Counts BWD from 10
three, two, one	five, four, three, two, one	ten, nine, eight, seven, six, five, four, three, two, one

Subitizing *(immediate recognition of quantity - five frames, fingers, regular dot patterns, irregular dot patterns)*

Perceptually subitizes <u>regular</u> patterns within 5 *(Immediate recognition of quantity)*	**Perceptually** subitizes <u>irregular</u> patterns within 5 *(immediate recognition of quantity)*
<u>Example</u> quick images to support subitizing regular patterns	***<u>Example</u> quick images to support subitizing irregular patterns***
	

Symbolic Notation

Identifies numerals within 5	Identifies numerals within 10
	

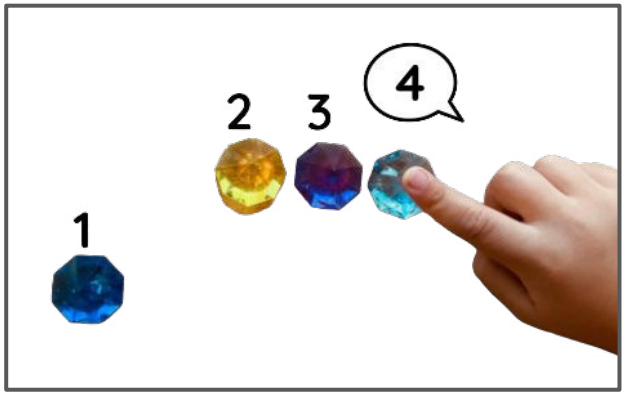
Count Objects to Determine Cardinality *(cardinality demonstrates understanding that the last number in the count is the quantity)*

Students are given amounts of discrete objects to determine the total quantity. All of the skills noted below are observable during a Counting Collection. Each understanding might develop at different times for each number range.

Counts objects within 5	Counts objects within 10

The following understandings develop at different times for each number range:

-1:1 correspondence (*each item gets one count)*
-Organizing (*keep track of what's been counted and what still needs to be counted without prompting*)
-Tracking methods (*the actual gesture of touching and counting)*
-Stable order (*correct number word sequence*)
-Cardinality (*last number in the count is the quantity)*
-Conservation of number (*quantity is the same regardless of arrangement - ex: objects lined up, then spread out, organized by 10 or not organized)*



1:1 Correspondence

Conservation of Number

Cardinality

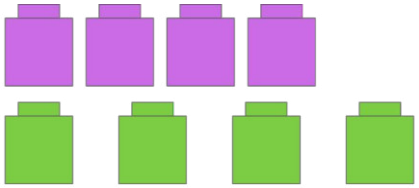
Ordering & Magnitude

For various quantities, students may compare by subitizing, matching (1:1) lining items up, or counting quantities. This concept is also impacted by conservation of number - consistent count regardless of orientation (*"It is still 4, the cubes are just spread out"*).

Compares quantities within 5

Five fingers are more than two fingers.

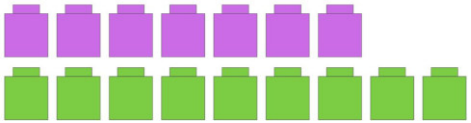
Two fingers are fewer than five fingers.



Both rows have the same amount of cubes. They both have four cubes.

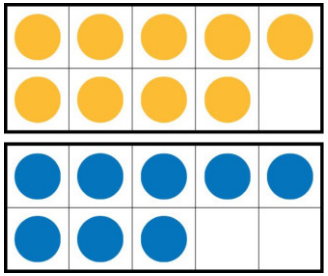
Compares quantities within 10

Seven cubes are fewer than nine cubes.



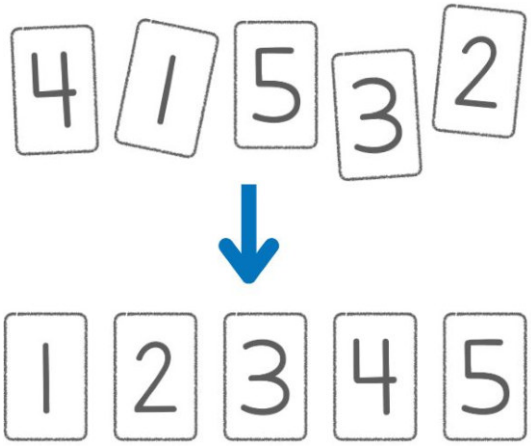
Nine cubes are more than seven cubes.

Nine dots are more than eight dots.

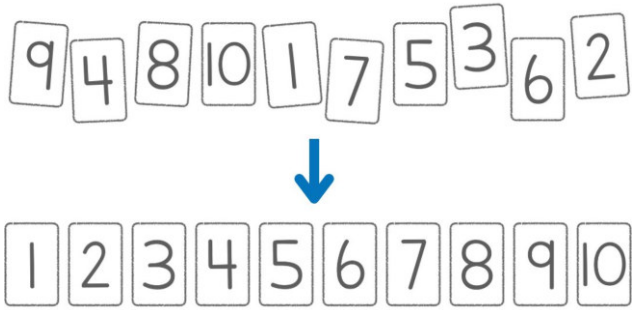


Eight dots are fewer than nine dots.

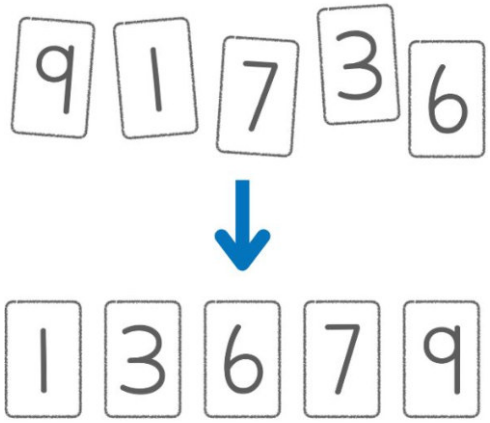
Orders numerals 1-5



Orders numerals 1-10



Orders non-sequential numerals within 10



Number

Kindergarten HLC

Understanding of number values and sequences to 20 *(counting, cardinality, and stable order)*
1:1 Correspondence **Comparing quantities**

September → **Kindergarten Learning Progressions** → June

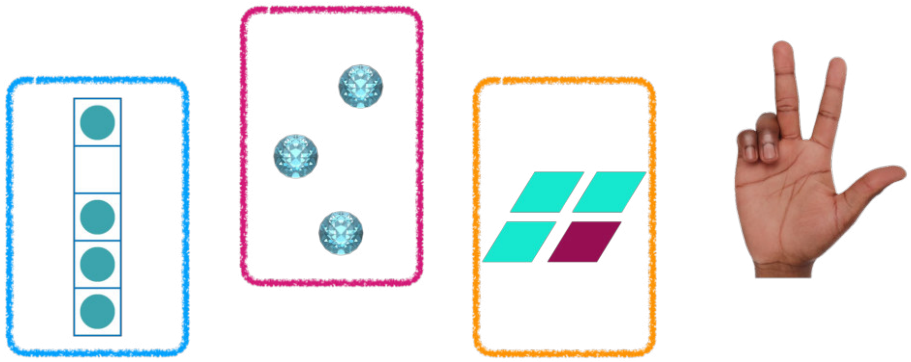
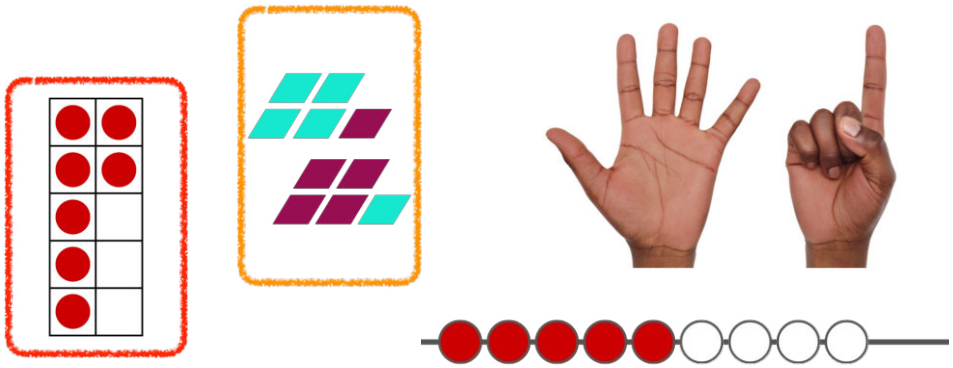
Students must use models to build understanding of the HLC and interact with a variety of contexts.

Rote Oral Count Sequence

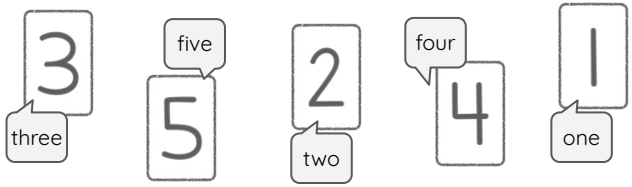
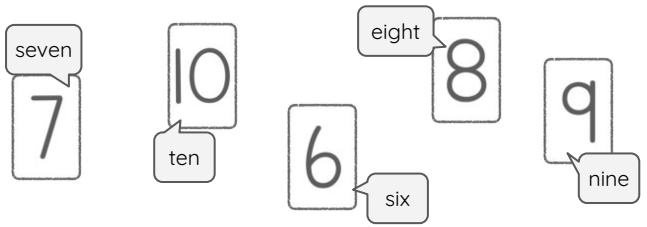
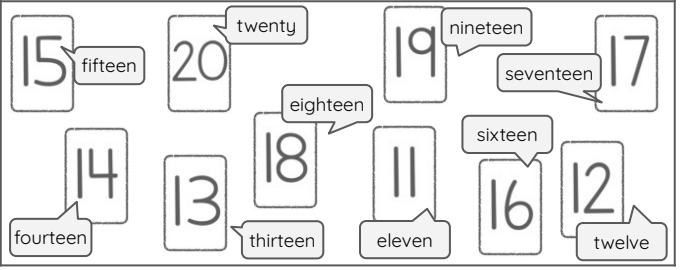
Counts Forward (FWD) from 1 to 5	Counts FWD from 1 to 10	Counts FWD from 1 to 20	Counts FWD within the range 1-20 starting at any number
one, two, three, four, five	one, two, three, four, five, six, seven, eight, nine, ten	one, two, three, four, five, six, seven, eight, nine, ten, eleven, twelve, thirteen, fourteen, fifteen, sixteen, seventeen, eighteen, nineteen, twenty	**Examples only:** three, four, five, six, seven, eight, nine eight, nine, ten, eleven, twelve, thirteen, fourteen, fifteen fifteen, sixteen, seventeen, eighteen, nineteen, twenty

Counts Backward (BWD) from 3	Counts BWD from 5	Counts BWD from 10	Counts BWD from 20	Counts BWD within the range 1-20, starting at any number
three, two, one	five, four, three, two, one	ten, nine, eight, seven, six, five, four, three, two, one	twenty, nineteen, eighteen, seventeen, sixteen, fifteen, fourteen, thirteen, twelve, eleven, ten, nine, eight, seven, six, five, four, three, two, one	**Examples only:** eleven, ten, nine, eight, seven twenty, nineteen, eighteen, seventeen, sixteen seventeen, sixteen, fifteen, fourteen, thirteen

Subitizing *(immediate recognition of quantity - five and ten frames, fingers, regular dot patterns, irregular dot patterns)*

Perceptually subitizes within 5 *(Immediate recognition of quantity)*	**Conceptually** subitizes within 10 *(Quickly compose greater quantities by "seeing" and combining smaller parts - part/part/whole and/or decomposing/composing)*
***Examples* of quick images to support perceptual subitizing** 	***Examples* of quick images to support conceptual subitizing** 

Symbolic Notation Reversals in numeral formation are expected at this developmental stage, but transpositions (eg., 71 for 17) are an indicator of a misconception and may interfere with representing quantities.

Identifies numerals within 5	Identifies numerals within 10	Identifies numerals within 20
		
Writes numerals within 5	Writes numerals within 10	Writes numerals within 20

Count Objects to Determine Cardinality *(cardinality demonstrates understanding that the last number in the count is the quantity)*

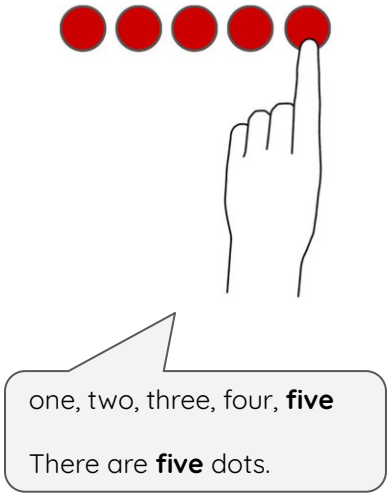
Students are given amounts of discrete objects to determine the total quantity. All of the skills noted below are observable during a Counting Collection. Each understanding might develop at different times for each number range.

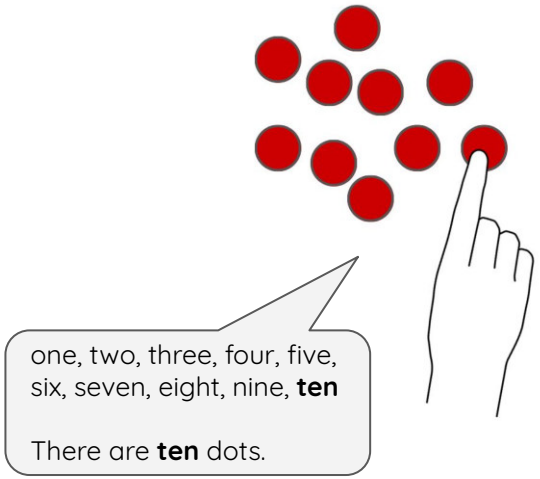
Counts objects within 5	Counts objects within 10	Counts objects within 20

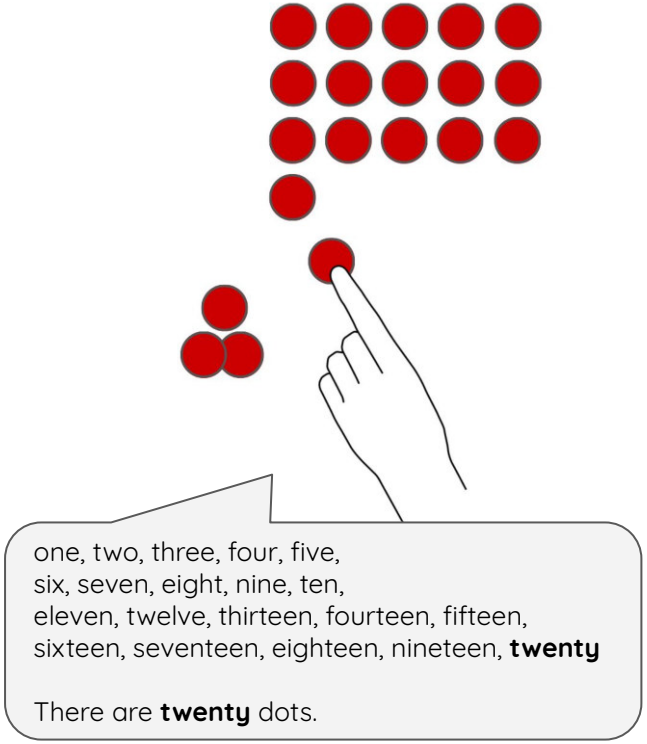
The following understandings develop at different times for each number range:

-1:1 correspondence (*each item gets one count)*
-Organizing (*keep track of what's been counted and what still needs to be counted without prompting*)
-Tracking methods (*the actual gesture of touching and counting)*
-Stable order (*correct number word sequence*)
-Cardinality (*last number in the count is the quantity)*
-Conservation of number (*quantity is the same regardless of arrangement - ex: objects lined up, then spread out, organized by 10 or not organized)*

***Examples* of counting collections**







Ordering & Magnitude

For various quantities, students may compare by subitizing, matching (1:1) lining items up, or counting quantities. This concept is also impacted by conservation of number - consistent count regardless of orientation (*"It is still 4, the cubes are just spread out"*).

Compares quantities within 5	Compares quantities within 10	Compares quantities within 20
Five fingers are more than two fingers. 5 is greater than 2. 2 is less than 5. Two fingers are fewer than five fingers.	Nine dots are more than eight dots. 9 is greater than 8. 8 is less than 9. Eight dots are fewer than nine dots.	Eighteen beads are more than sixteen beads. 18 is greater than 16. Sixteen beads are fewer than eighteen beads. 16 is less than 18.
Orders numerals 1-5	Orders numerals 1-10 (sequential or nonsequential)	Orders numerals 1-20 (sequential)
4 1 5 3 2 1 2 3 4 5	9 4 8 10 1 7 5 3 6 2 1 2 3 4 5 6 7 8 9 10 8 10 7 5 2 2 5 7 8 10	13 14 15 12 16

ALL LEARNERS NETWORK

Grade One HLC Learning Progressions

Additive Reasoning

Grade One HLC

Understanding of number values and sequences to 120 *(cross century, cross decade)*
Understanding place value when adding and subtracting numbers within 100 *(in context and in equations)*

September → **Grade One Learning Progressions** → June

Students must use models to build understanding of the HLC and interact with a variety of contexts.

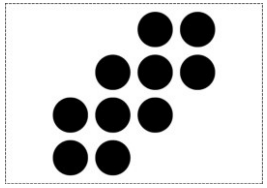
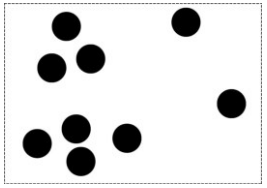
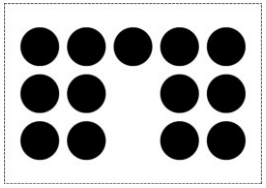
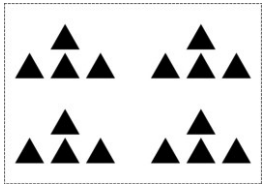
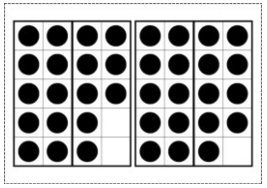
Rote Oral Count Sequence Teachers need to purposefully choose a variety of number ranges including opportunities to practice teen numbers, crossing decades, and centuries. This information is often best collected in student interviews checking on clusters of 5 numbers at various starting points.

Counts Forward (FWD) and Backward (BWD) within the range **1-30** starting at any number	Counts FWD and BWD within the range **1-50** starting at any number	Counts FWD and BWD within the range **1-100** starting at any number Skip counts by 10s FWD and BWD within the range 1-100 on decade.	Counts FWD and BWD within the range **1-120** starting at any number Skip counts by 10s FWD and BWD within the range **1-120** starting at any number.

Subitizing *(immediate recognition of quantity - ten and twenty frames, fingers, regular dot patterns)*

Conceptual subitizing within 20 (*quickly composing greater quantities by seeing and combining smaller parts and using groups of ten)*
This connects to an understanding of part/part/total and/or decomposing and recomposing.

***Examples* of quick images to support conceptual subitizing**

Symbolic Notation Reversals in numeral formation are expected at this developmental stage, but transpositions (eg., 71 for 17) are an indicator of a misconception and may interfere with representing quantities.

Identifies and writes numerals within 20	Identifies and writes numerals within 100	Identifies and writes numerals within 120

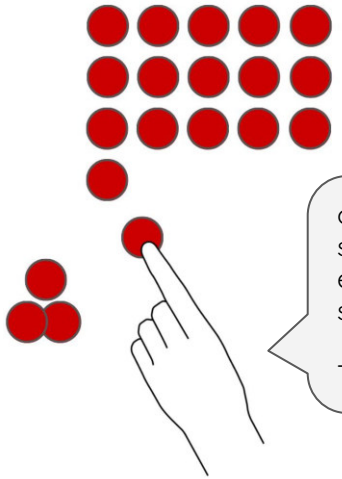
Counting Collections to Build Place Value Understanding

Students must use models to build understanding along this trajectory and interact with a variety of contexts for counting. Models should support students developing understanding of the magnitude of digits in their place values. Students are given amounts of discrete objects to determine the total quantity. All of the skills noted below are observable during a Counting Collection. Each understanding might develop at different times for each number range. **Students must use models to build understanding of unitizing: 10 ones = 1 ten; 10 tens = 1 hundred.**

Counts objects within 20	Counts objects within 50 (using groups of ten)	Counts objects within 100 (using groups of ten)	Counts objects within 120 (using groups of ten)

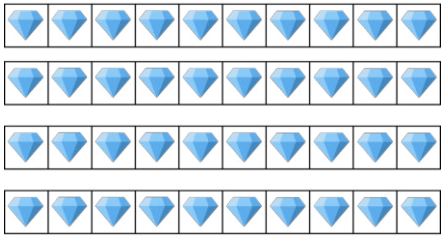
-1:1 correspondence (*each item gets one count)*
-Organizing (*keep track of what's been counted and what still needs to be counted without prompting*)
-Tracking and recording methods (*organizing, grouping and recording)*
-Stable order (*correct number word sequence*)
-Conservation of number (*quantity is the same regardless of arrangement - ex: objects lined up, then spread out, organized by 10 or not organized)*

***Examples* of counting collections**



one, two, three, four, five,
six, seven, eight, nine, ten,
eleven, twelve, thirteen, fourteen, fifteen,
sixteen, seventeen, eighteen, nineteen, **twenty**

There are **twenty** dots.

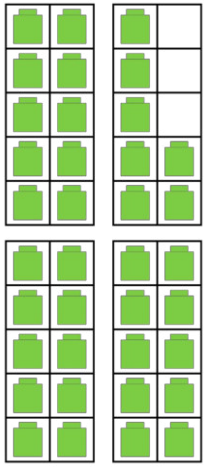


Ten, twenty, thirty, **forty**

There are **forty** gems.

ten, twenty, thirty,
thirty-one, thirty-two,
thirty-three, thirty-four,
thirty-five, thirty-six,
thirty- seven

There are **thirty-seven** cubes.

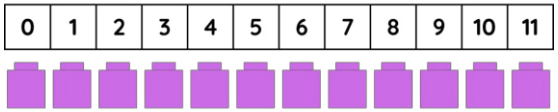


1 ten, 2 tens, 3 tens
and 7 ones is
thirty-seven

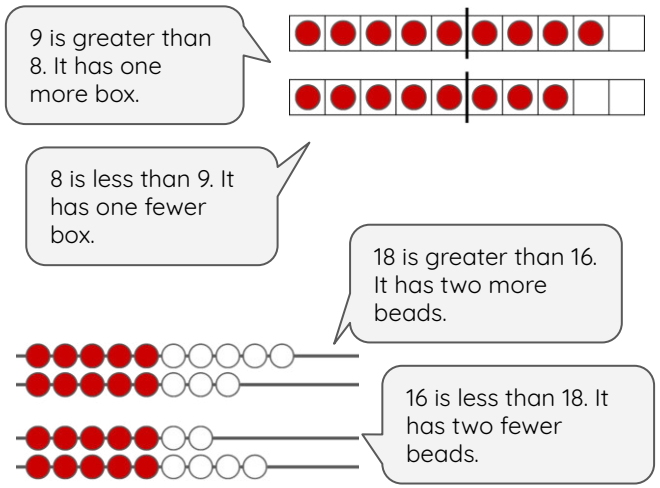
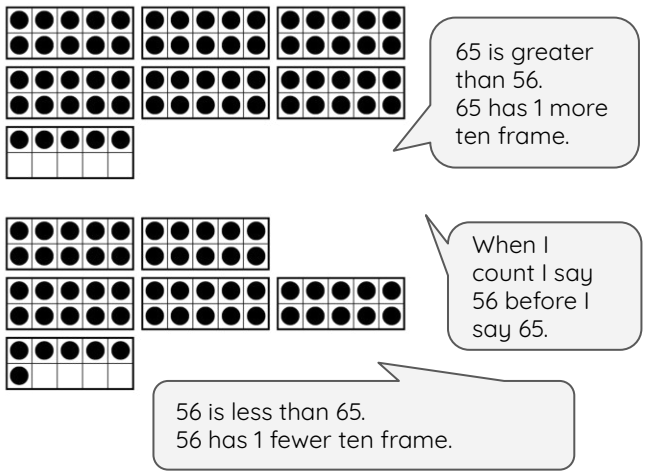
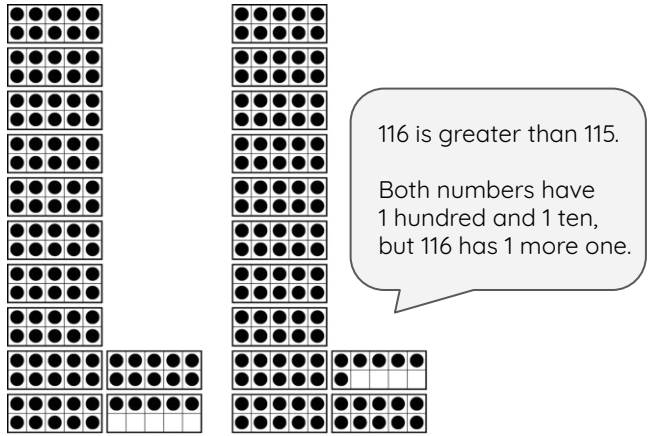
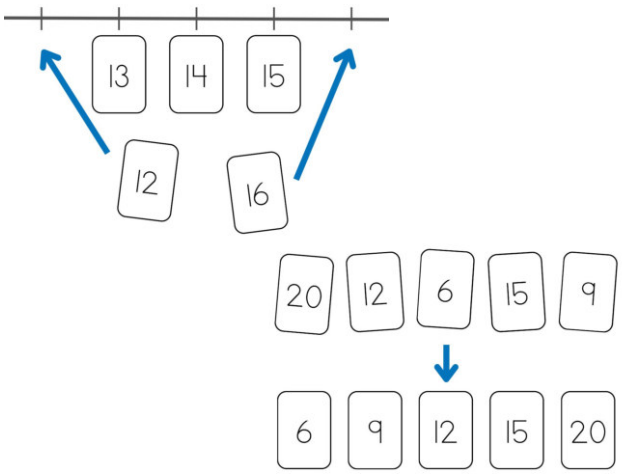
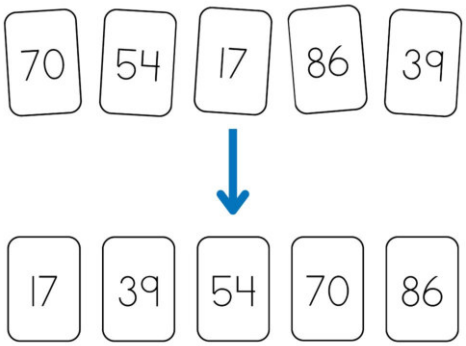
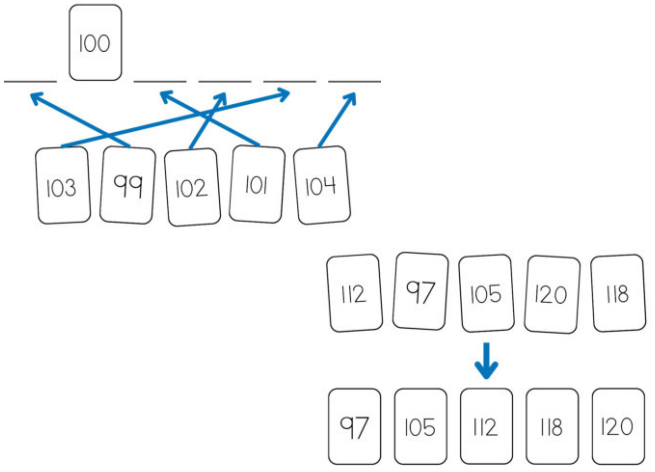
There are
thirty-seven cubes.

one, two, three, four, five,
six, seven, eight, nine, ten,
eleven

There are **eleven** cubes.

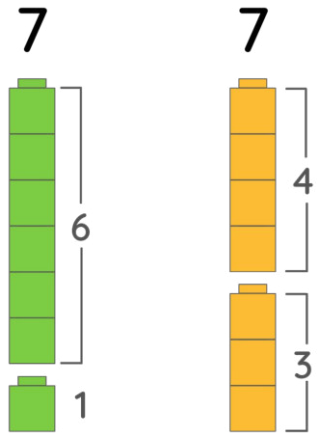
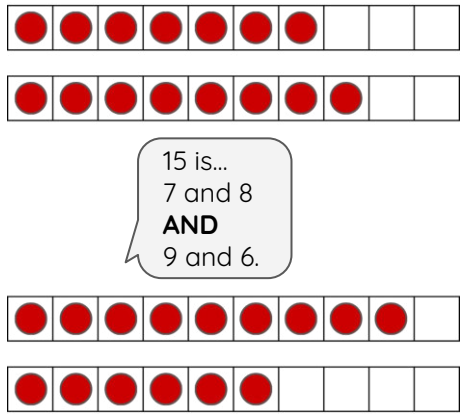
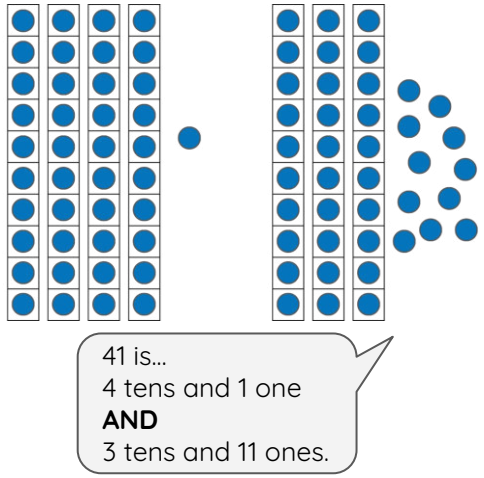
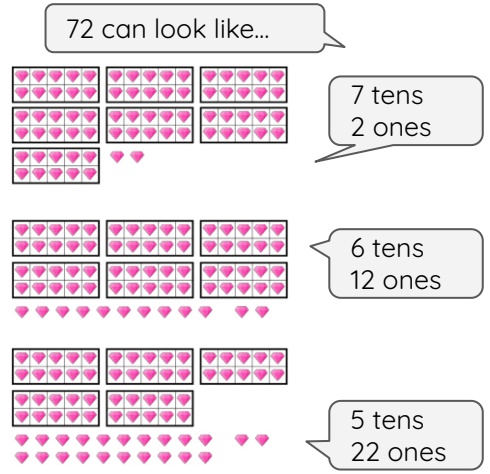


Ordering & Magnitude

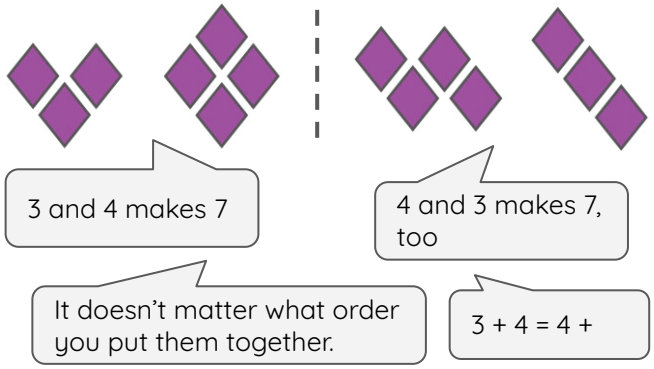
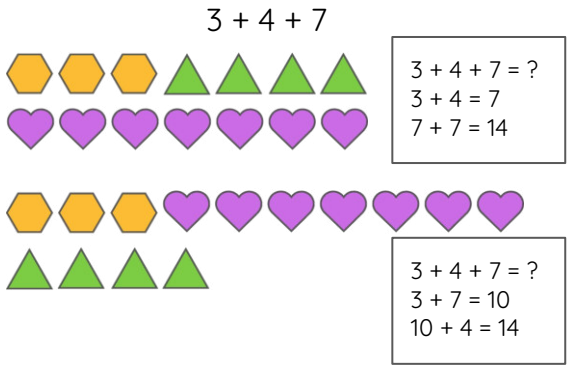
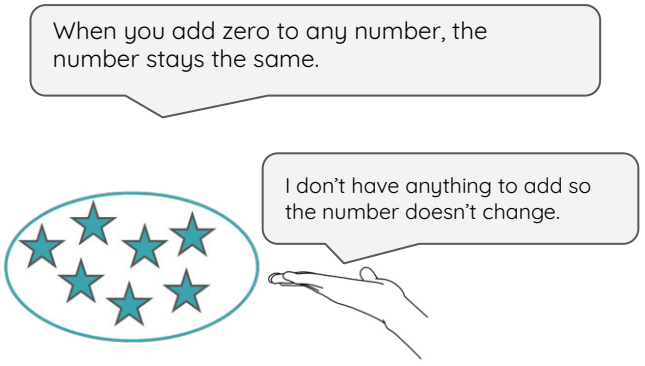
Compares quantities within 20 by using items or visuals - using perception and/or counting	Compares quantities within 100 by using models - using references to counting or how many more/less or by using place value	Compares quantities within 120 by using models - using place value language, including knowledge of tens and ones
		
Orders numerals, sequential and nonsequential, within 20	Orders numerals, sequential and nonsequential, within 100	Orders numerals, sequential and nonsequential, within 120
		

Operations: Addition and Subtraction Students must use models to build understanding along this trajectory and interact with a variety of contexts for addition and subtraction. Models should support students developing understanding of the magnitude of digits in their place values.

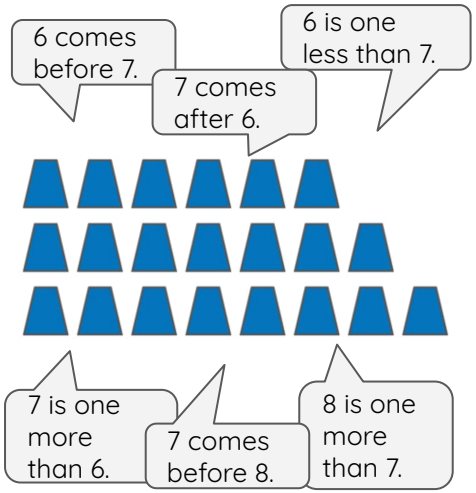
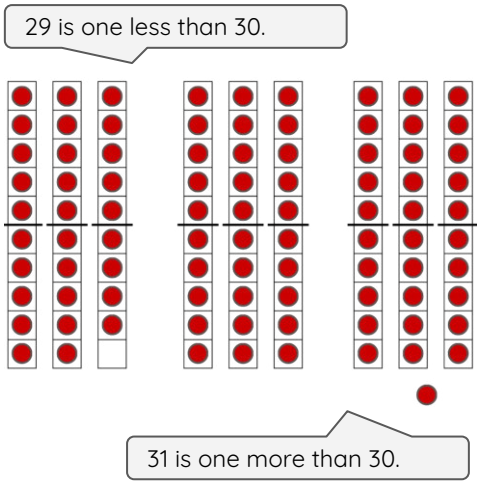
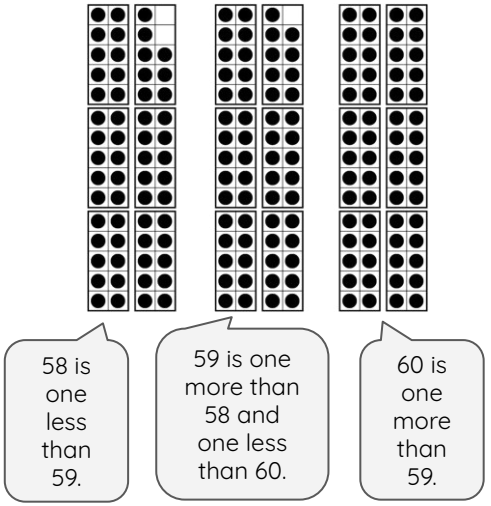
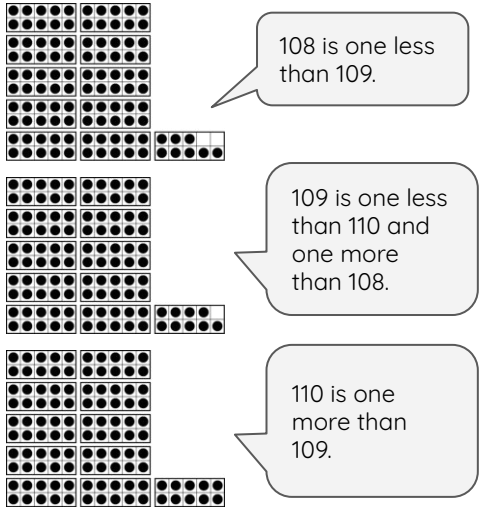
Composition, Decomposition Students must use models to build understanding and flexibility when composing and decomposing quantities.

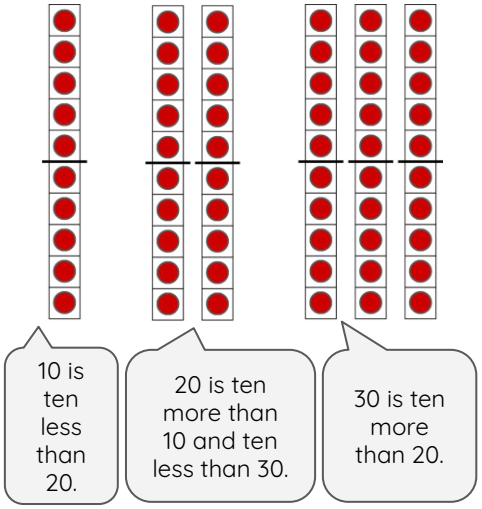
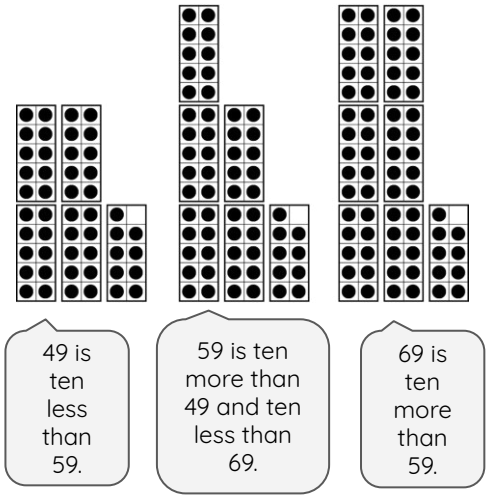
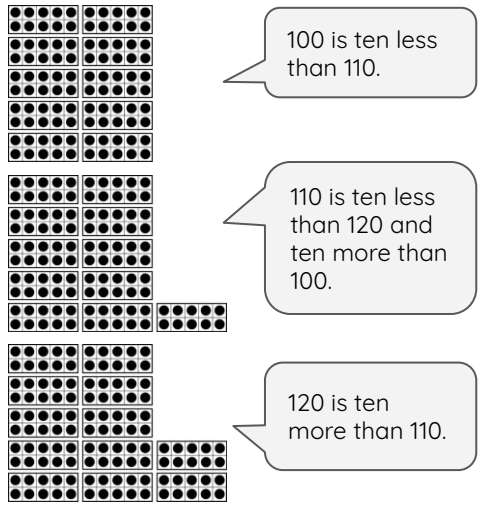
All numbers within the range 1-10	All numbers within the range 1-20	All numbers within the range 1-50	All numbers within the range 1-100
			

Properties of Addition These properties are investigated throughout the year with different numbers and problem situations. *The sequence of how the properties appear below does not suggest the order in which to explore them.* Many times the properties can be explored simultaneously with student work.

Commutative Property	Associative Property	Identity Property
		

Place Value - Building Understanding Students must use models to build understanding along this trajectory and interact with a variety of contexts for addition and subtraction. Models should support students developing understanding of the magnitude of digits in their place values.

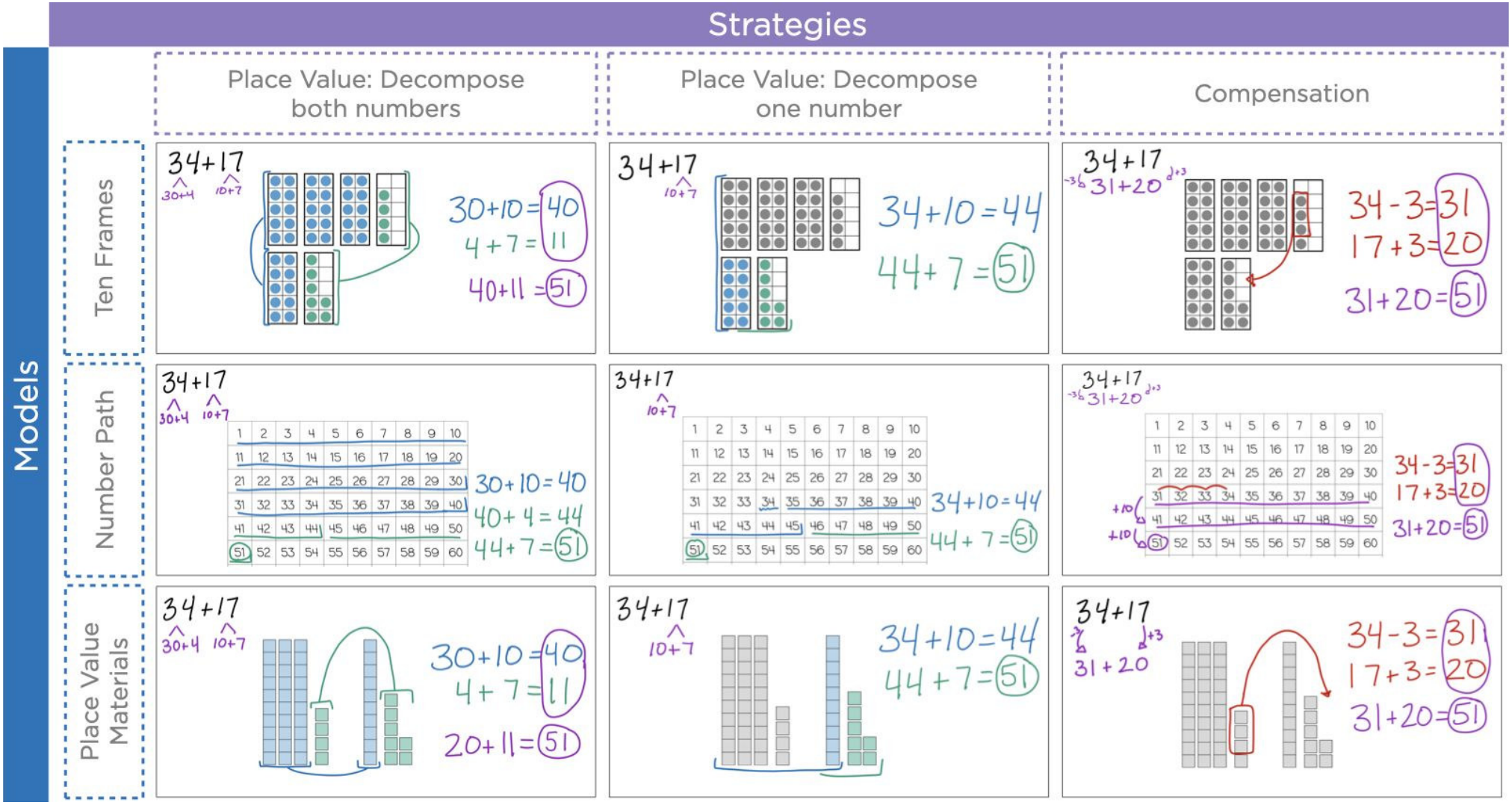
Models the number 1 more/1 less within 20 (connect to before/after)	Models the number 1 more/1 less within 50	Models the number 1 more/1 less within 100	Models the number 1 more/1 less within 120
			

Models the number 10 more/10 less within 50	Models the number 10 more/10 less starting at any number within 100	Models the number 10 more/10 less starting at any number within 120
		

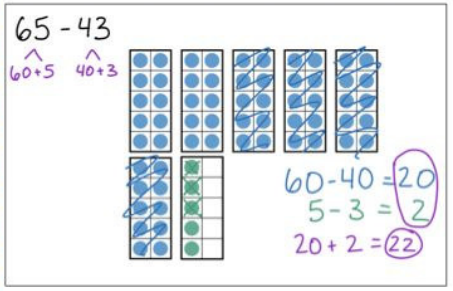
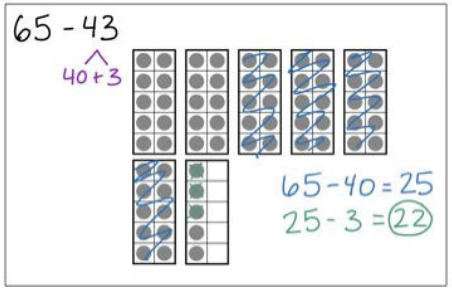
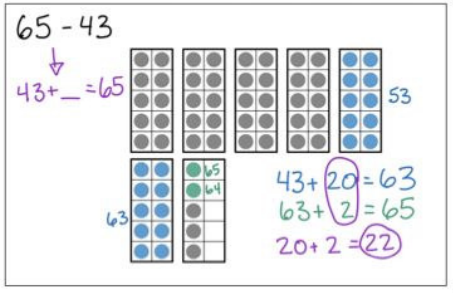
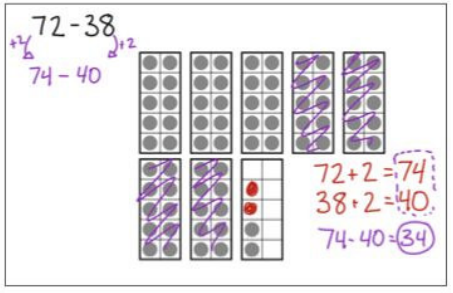
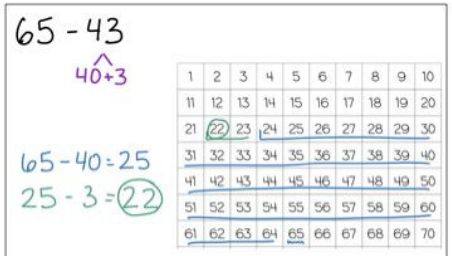
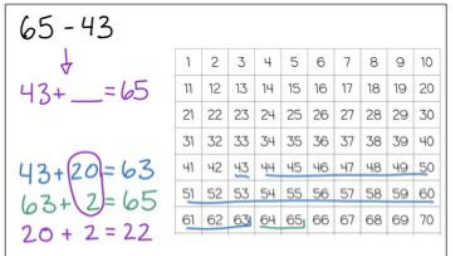
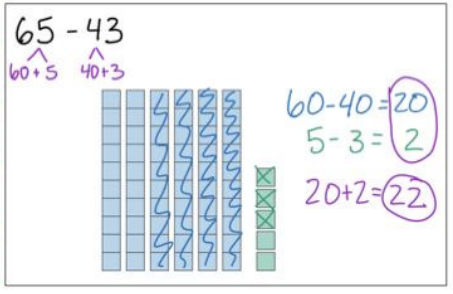
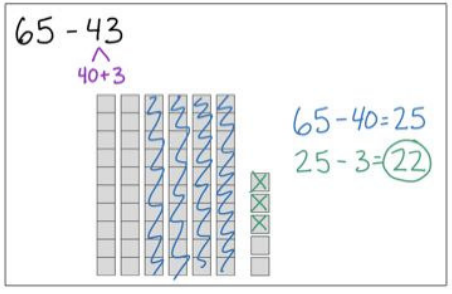
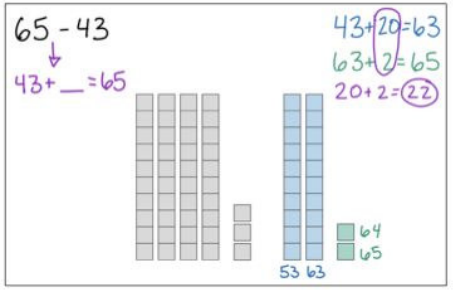
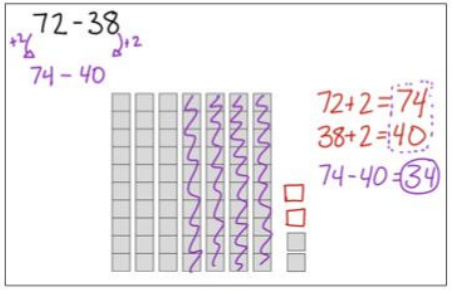
Use Place Value to compose, decompose and recompose

Decompose both numbers to add and subtract, decompose one number to add and subtract, recompose like units, missing addend, compensation
There is an explicit connection between counting and addition (i.e. counting 10 more is the same as adding 10, counting back 10 is the same as subtracting 10).

Models & Strategies for Addition



Models & Strategies for Subtraction

Models	Strategies: Place Value: Decompose both numbers	Place Value: Decompose one number	Missing Addend	Compensation
Ten Frames				
Number Path	Number Paths are not an appropriate model for this strategy.			Number Paths are not an appropriate model for this strategy.
Place Value Materials				

Additive Reasoning

Grade Two HLC

Use place value understanding to add and subtract numbers accurately, flexibly, efficiently, and strategically within 1,000
(in context and in equations) **(NO standard algorithm)**

September → **Grade Two Learning Progressions** → June

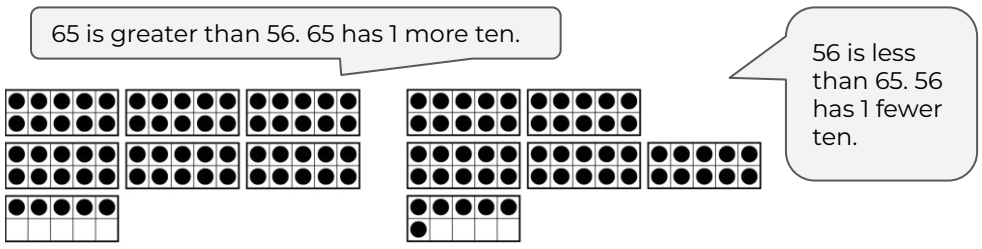
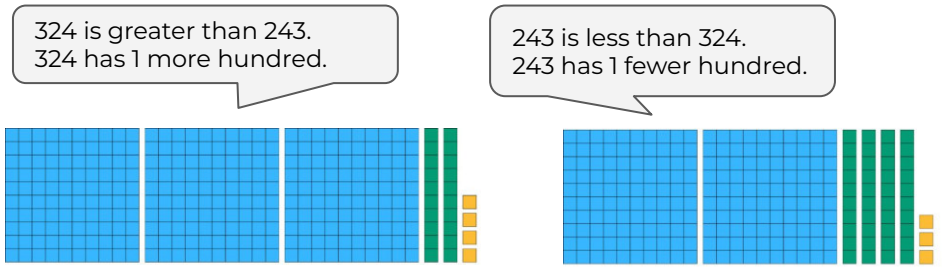
Students must use models to build understanding of the HLC and interact with a variety of contexts.

Rote Oral Count Sequence *(rote counting from 1; rote counting from any start number)*

Teachers need to purposefully choose a variety of number ranges including opportunities to practice teen numbers, crossing decades, and centuries. This information is often best collected in student interviews checking on clusters of 5 numbers at various starting points.

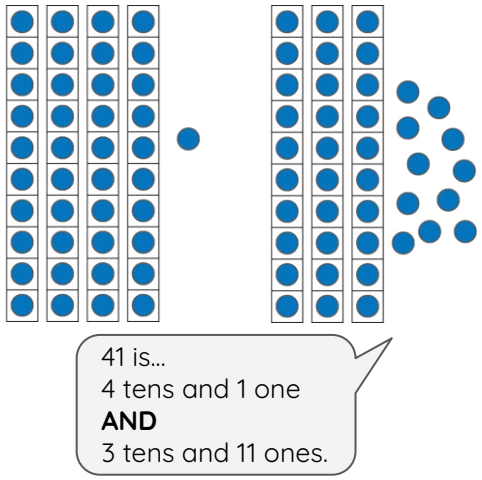
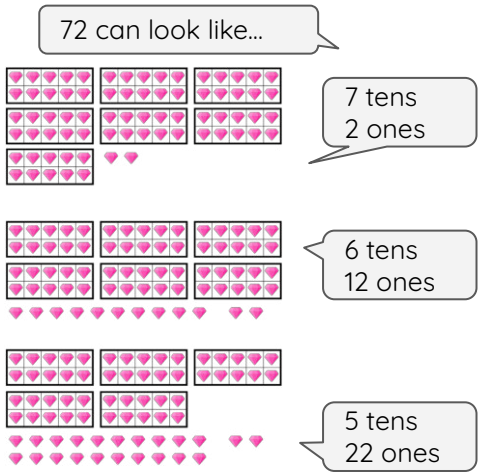
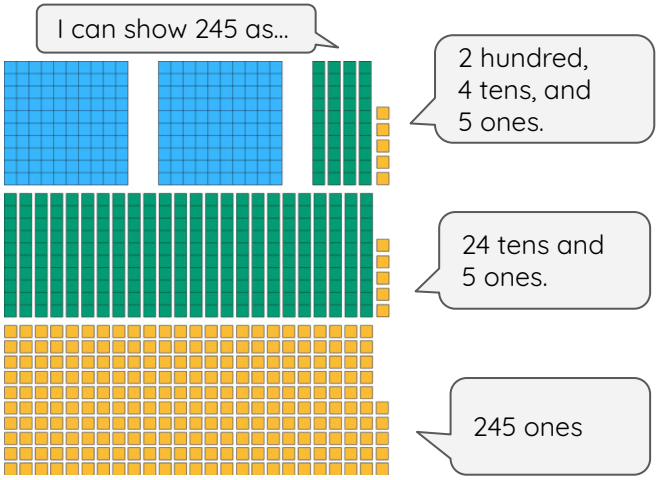
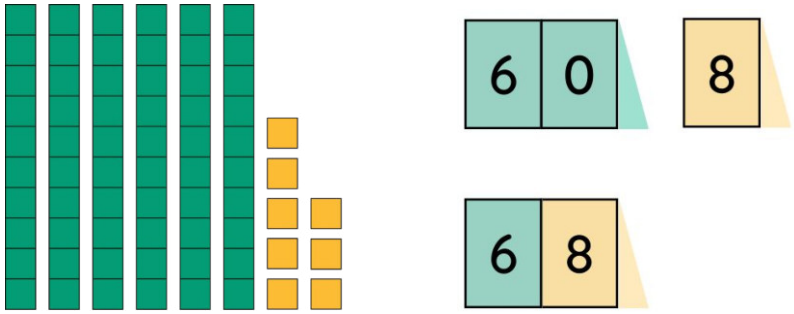
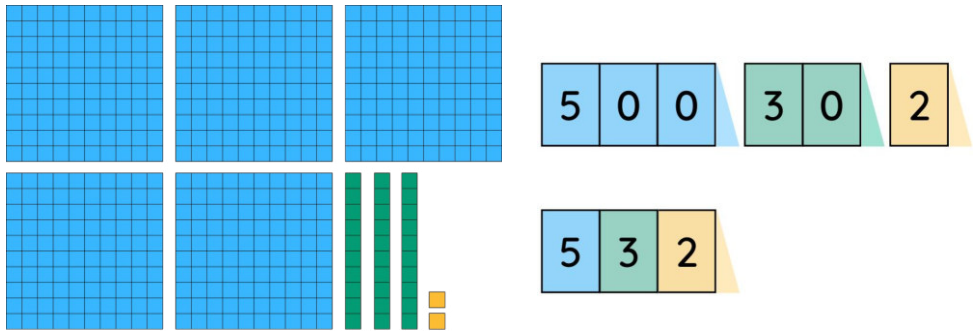
Counts Forward (FWD) and Backward (BWD) within the range **1-120** starting at any number	Counts FWD and BWD within the range **1-220** starting at any number	Counts FWD and BWD within the range **1-500** starting at any number	Counts FWD and BWD within the range **1-1000** starting at any number
Skip counts FWD and BWD by 10s **starting at any number** within the range **1-120**	Skip counts FWD and BWD by 10s **on decade** within the range **1-1000**	Skip counts FWD and BWD by 10s starting at any number within the range **1-500**	Skip counts FWD and BWD by 10s starting at any number within the range **1-1000**
Skip counts FWD and BWD by 100s starting on century within the range **1-1000**		Skip counts FWD and BWD by 100s starting at any number within the range **1-1000**	

Ordering & Magnitude

Uses place value understanding to compare 2-digit numbers.	Uses place value understanding to compare 3-digit numbers.
	

Operations: Addition and Subtraction Students must use models to build understanding along this trajectory and interact with a variety of contexts for addition and subtraction. Models should support students developing understanding of the magnitude of digits in their place values.

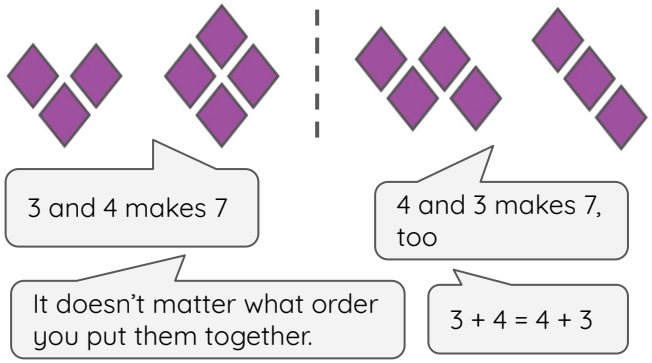
Composition, Decomposition Students must use models to build understanding and flexibility when composing and decomposing quantities. Students must use models to build understanding of unitizing: 10 ones = 1 ten; 10 tens = 1 hundred, etc. as well as *equivalent* representations of a specific quantity (i.e 126 is simultaneously 126 ones; 12 tens and 6 ones; 1 hundred, 2 tens, and 6 ones; 1 hundred and 26 ones; 11 tens and 16 ones; 9 tens and 36 ones; etc.)

All numbers within the range 1-50	All numbers within the range 1-100	All numbers within the range 1-1000
		
Builds and tells the value of the digits in any 2-digit number. Decompose any 2-digit number into its place value parts.	Builds and tells the value of the digits in any 3-digit number. Decompose any 3-digit number into its place value parts.	
		

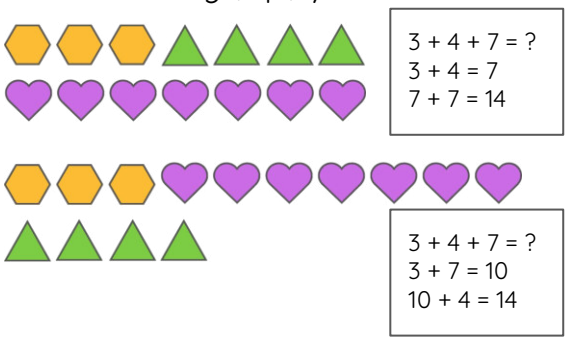
Properties of Addition

These properties are investigated throughout the year with different numbers and problem situations. The sequence of how the properties appear below does not suggest the order in which to explore them. Many times the properties can be explored simultaneously with student work.

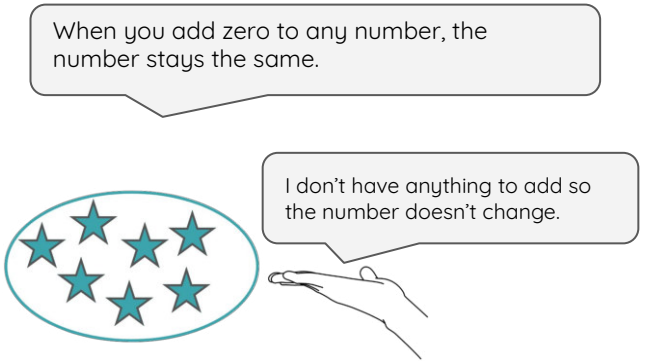
Commutative Property



Associative Property



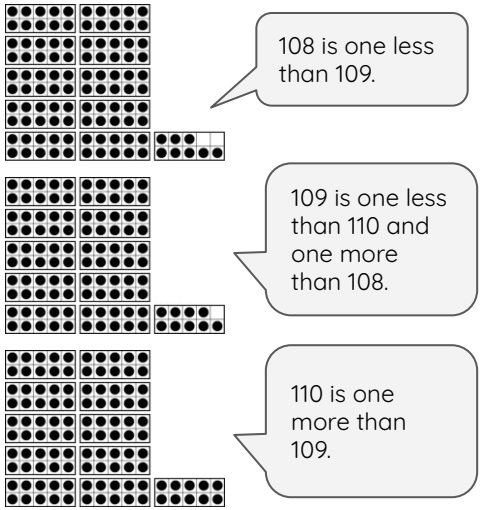
Identity Property



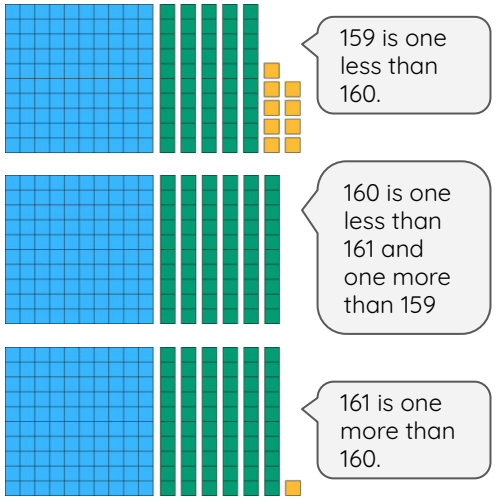
Place Value - Building Understanding

Students must use models to build understanding along this trajectory and interact with a variety of contexts for addition and subtraction. Models should support students developing understanding of the magnitude of digits in their place values.

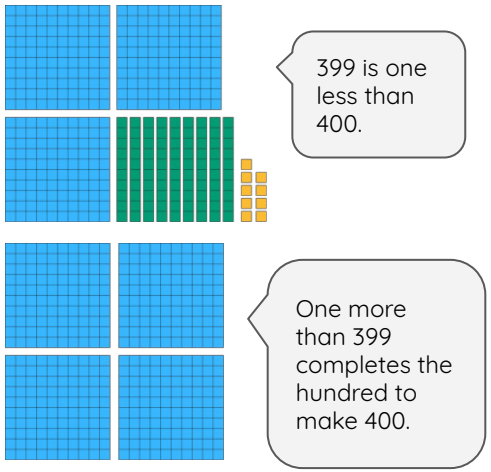
Models the number 1 more/1 less within 120



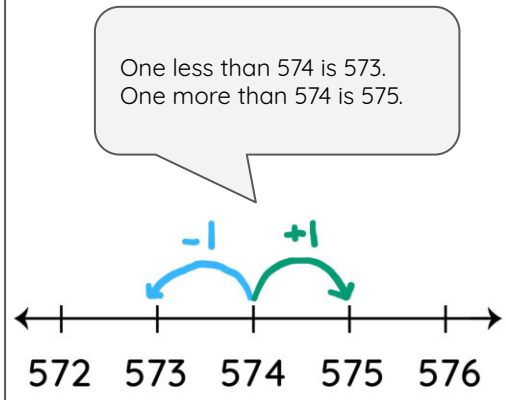
Models the number 1 more/1 less within 220



Models the number 1 more/1 less within 500



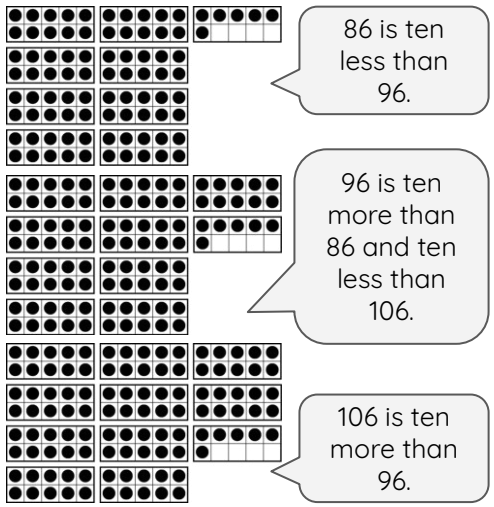
Models the number 1 more/1 less within 1000



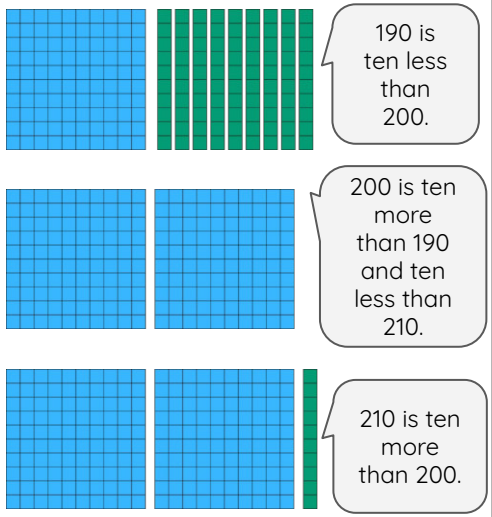
This section continued on next page.

Place Value - Building Understanding *(cont.)*

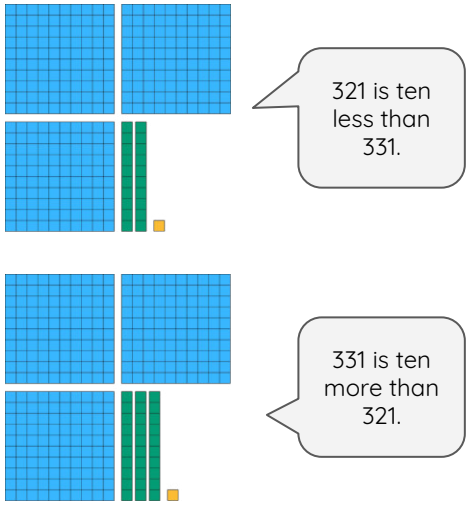
Models the number 10 more/10 less from any number within 120



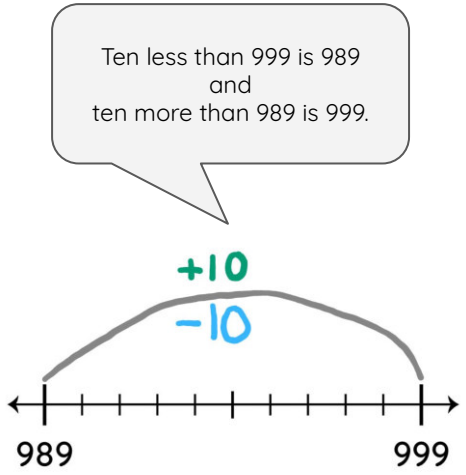
Models the number 10 more/10 less from any number within 220



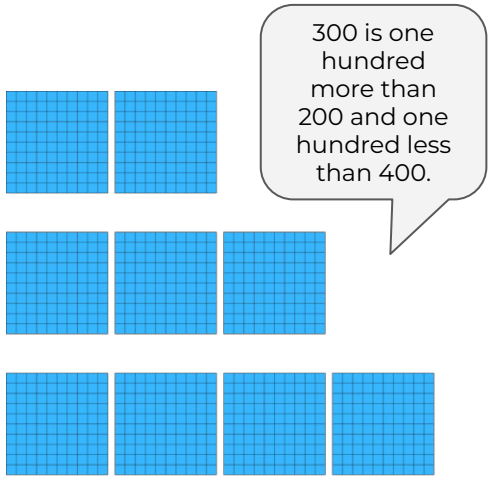
Models the number 10 more/10 less from any number within 500



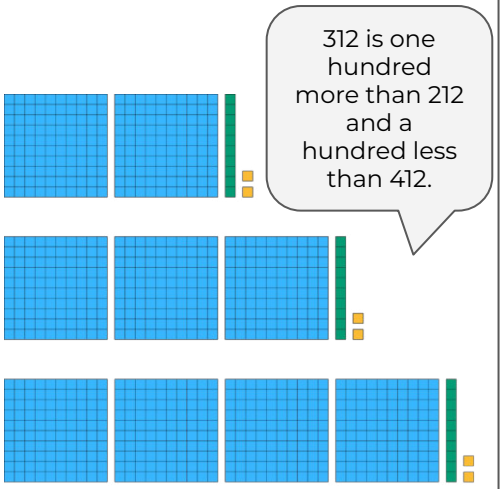
Models the number 10 more/10 less from any number within 1000



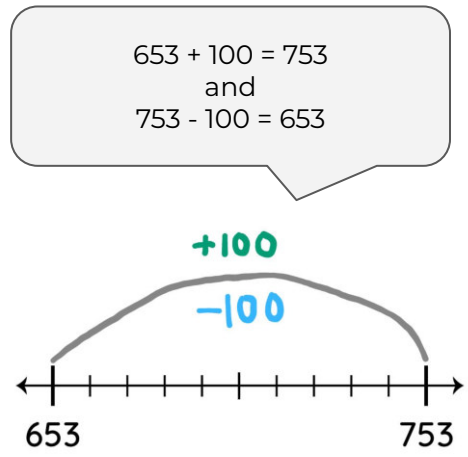
Models the number 100 more/100 less from any century within 1000



Models the number 100 more/100 less from any number within 500

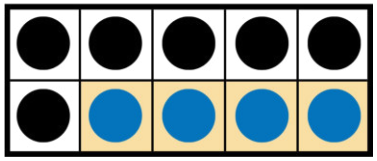
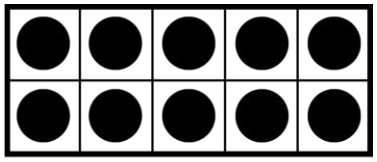
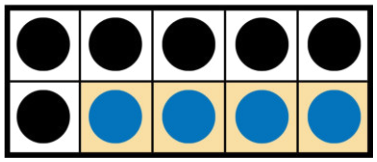
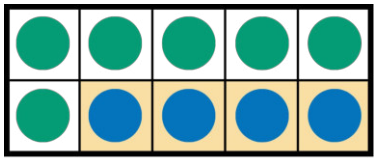
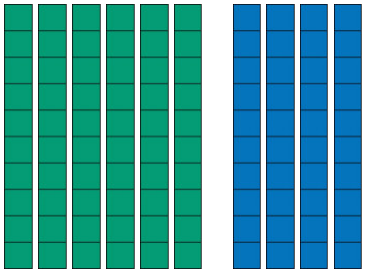
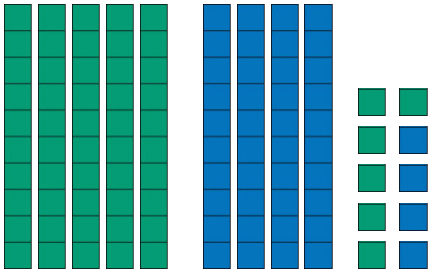
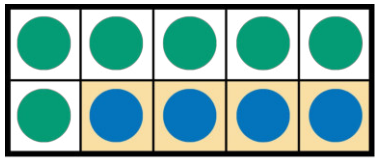
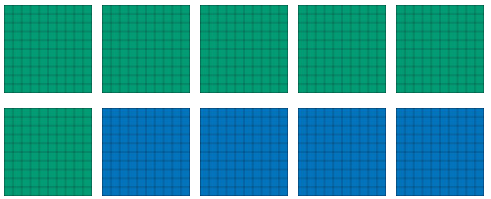
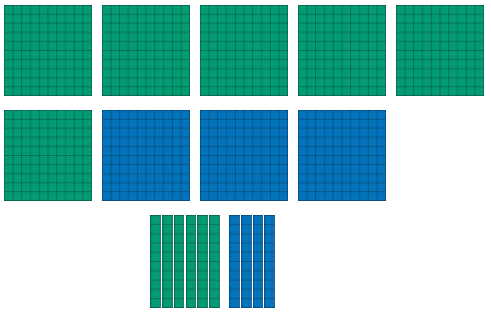


Models the number 100 more and 100 less from any number within 1000



Developing and Extending Fact Fluency

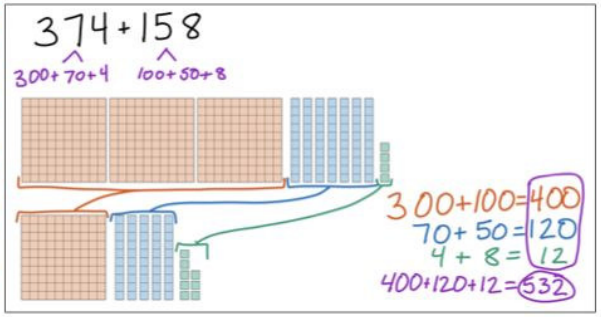
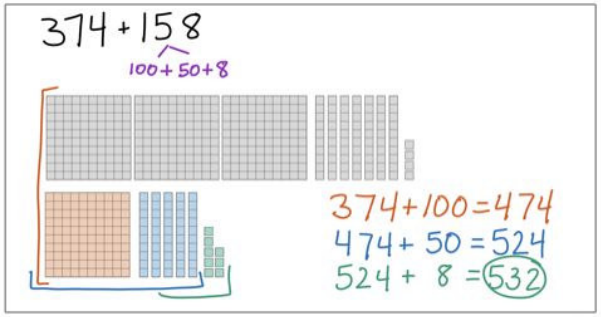
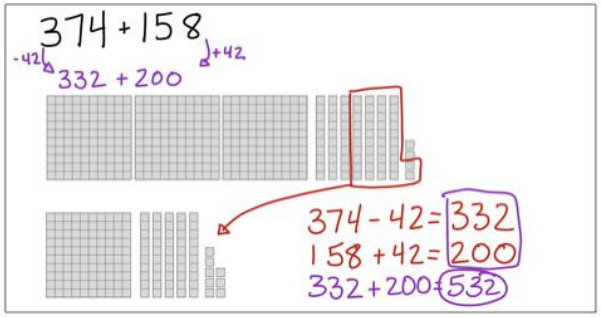
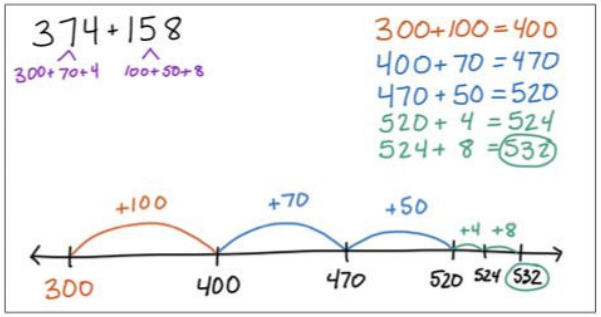
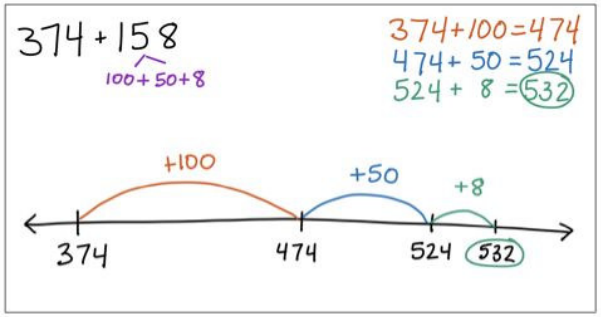
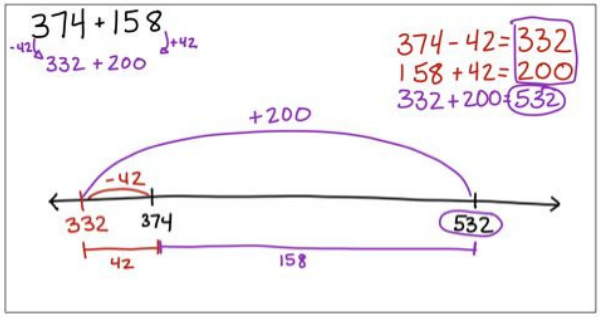
Students use relational thinking to develop fact fluency within 10 and then extend those fact patterns to greater numbers.

Uses understanding of combinations to 10 to find combinations to 20.	Uses understanding of combinations to 10 to find multiple of 10s partners to 100.	Uses understanding of combinations to 10 to find multiple of 100s partners to 1000.	Uses understanding of combinations to 1s, 10s, 100s to add any numbers within 1000.
 6 + 4 = 10 so 16 + 4 = 20  	 6 + 4 = 10 so 60 + 40 = 100  **56 + ___ = 100** 50 + 40 = 90 6 + 4 = 10 90 + 10 = 100 **56 + 44 = 100** 	 6 + 4 = 10 so 600 + 400 = 1000  **560 + ___ = 1000** 500 + 400 = 900 60 + 40 = 100 900 + 100 = 1000 **560 + 440 = 1000** 	**See model/strategy charts on the following pages for examples of adding and subtracting within 1000.**

Use Place Value to compose, decompose and recompose

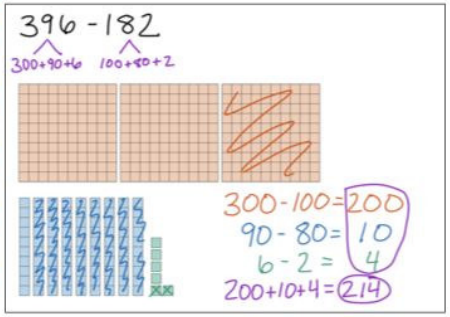
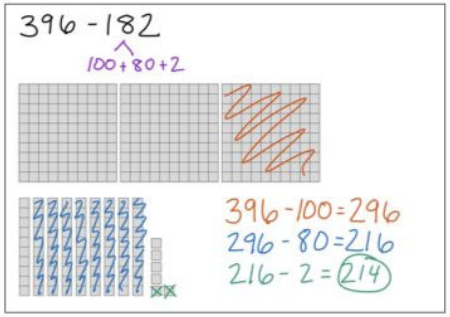
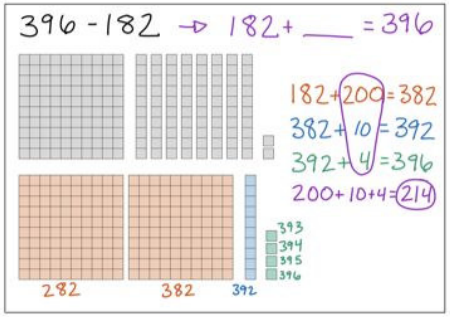
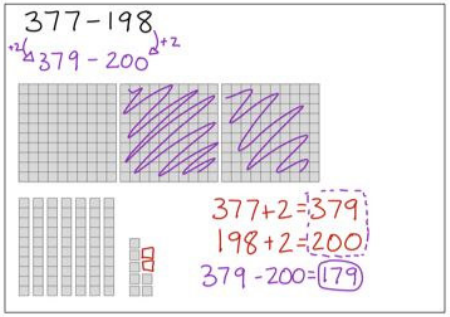
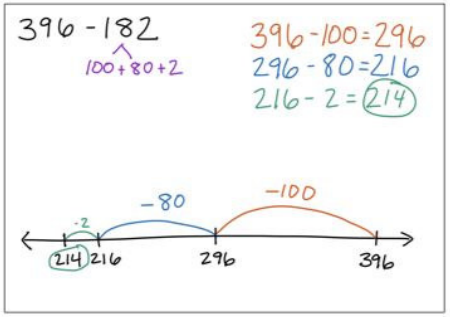
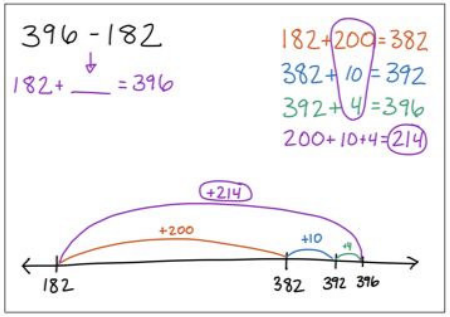
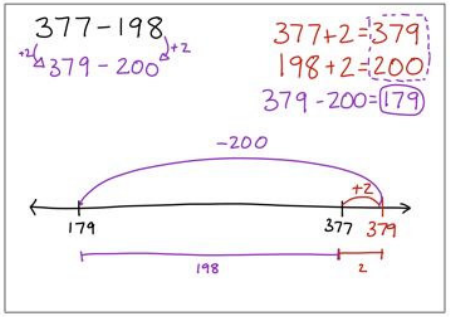
Decompose both numbers to add and subtract, decompose one number to add and subtract, recompose like units, missing addend, compensation
There is an explicit connection between counting and addition (i.e. counting 10 more is the same as adding 10, counting back 10 is the same as subtracting 10).

Models & Strategies for Addition

Models	Place Value: Decompose both numbers	Place Value: Decompose one number	Compensation
Ten Frames	Ten Frames are not an efficient model for 3-digit computation. See Grade 1 Progression for examples with 2-digit numbers.		
Place Value Materials			
Number Lines			

**We recommend starting with articulated number lines in Grade 2, and then connecting them to open number lines while moving from 2-digit to 3-digit computation.*

Models & Strategies for Subtraction

Models	Strategies: Place Value: Decompose both numbers	Place Value: Decompose one number	Missing Addend	Compensation
Ten Frames	Ten Frames are not an efficient model for 3-digit computation. See Grade 1 Progression for examples with 2-digit numbers.			
Place Value Materials				
Number Lines	Number Lines are not an appropriate model for this strategy.			

**We recommend starting with articulated number lines in Grade 2, and then connecting them to open number lines while moving from 2-digit to 3-digit computation.*

Multiplicative Reasoning

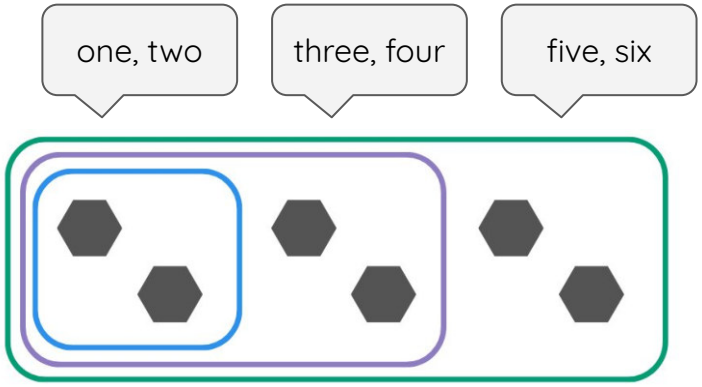
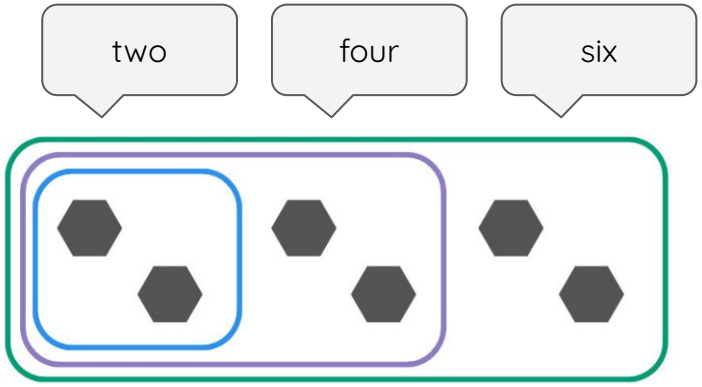
Grade Three HLC

Multiply and divide within 100 within context and with equations.

September → **Grade Three Learning Progressions** → June

Students must use models to build understanding along this trajectory and interact with a variety of contexts for multiplication and division. Models should support students' ability to unitize–understand a group or collection of items represents "one." (For example, one group of 5 consists of 5 individual items but is classified as one group.)

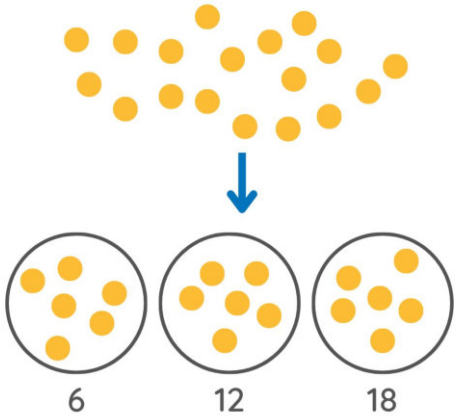
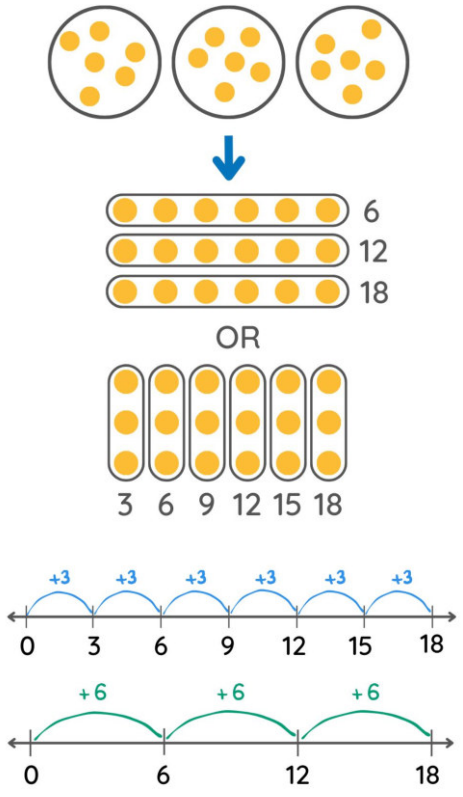
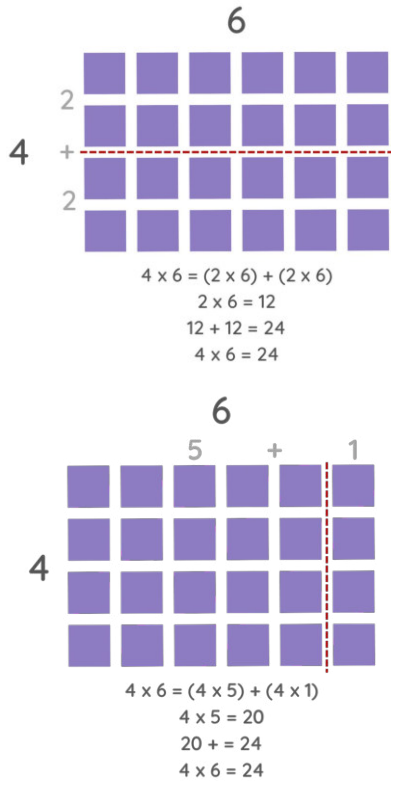
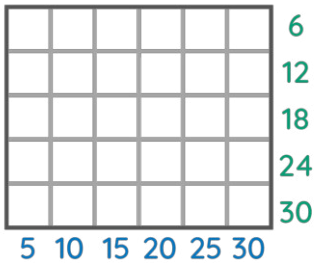
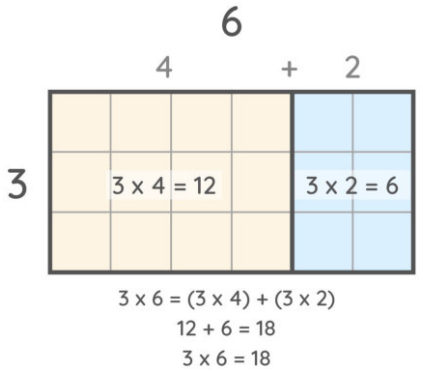
Counting by Equal Groups (Unitizing) to Build Multiplicative Understanding *(modeling and then counting by 1s or skip counting)*

Counts by ones in equal sized subgroups; counts individual objects within the group.	Skip counts the equal sized groups or uses repeated addition to tell the cumulative total of each group (no longer counts individual objects, but counts equal groups).
	

Operations: Multiplication and Division

Students must use models to build understanding along this trajectory and interact with a variety of contexts for multiplication and division. Models should support students developing understanding of the magnitude of digits in their place values. In Grades 1 and 2, students thought about place value as follows: 245 = 200 + 40 + 5. In Grades 3 and 4, place value understanding becomes multiplicative: 245 = 2(100) + 4(10) + 5(1)
Students also use relational thinking when composing, decomposing and recomposing

Multiplication - Composition and Decomposition

Organizes a collection into equal groups.	Organizes equal groups into rows and columns and skip counts by rows OR columns. Organizes equal groups as jumps on a number line.	Uses an array to decompose into smaller arrays.	Uses the area model for products to 100 (10x10) to understand length and width as dimensions that are 1x1 square units.
			**Skip counts rows or columns**  **Decomposes side lengths using the distributive property** 

Properties of Multiplication *These properties are investigated throughout the year with different numbers and problem situations. The sequence of how the properties appear below does not suggest the order in which to explore them. Many times the properties can be explored simultaneously with student work.)*

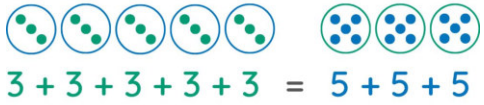
Identity Property

$7 \times 1 = 7$

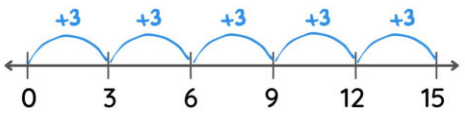
I have seven groups of one, which is the same as seven.

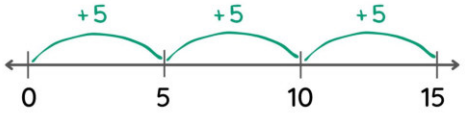
Commutative Property

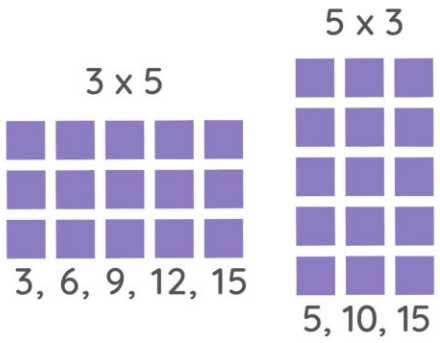
$3 \times 5 = 5 \times 3$



Three groups of five is the same value as five groups of three.





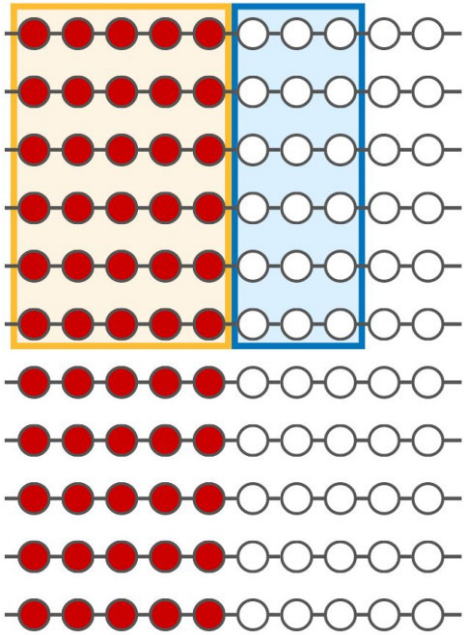


Distributive Property

$6 \times 8 = (6 \times 5) + (6 \times 3)$

Eight groups of six is the same value as six groups of five plus six groups of three.

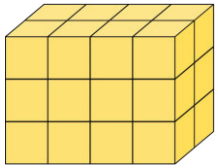
$30 + 18 = 48$



100 bead rack illuminates benchmark numbers

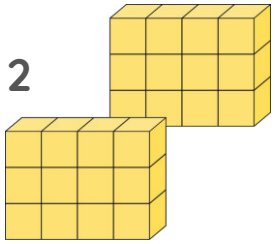
Associative Property

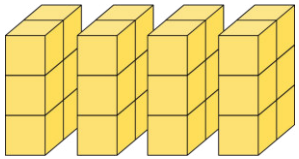
$4 \times 3 \times 2$



I can multiply the dimensions in any order to find the volume.

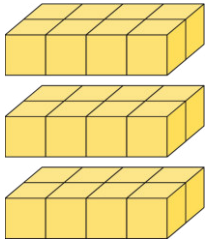
$(4 \times 3) \times 2$
12×2
24





$4 \times (3 \times 2)$
4×6
24

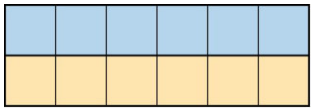
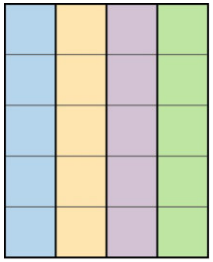
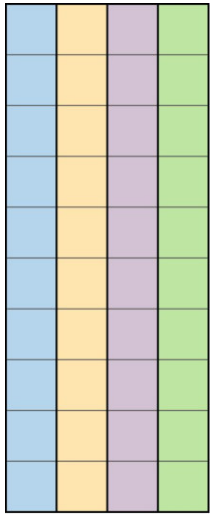
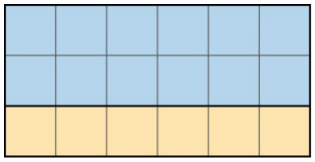
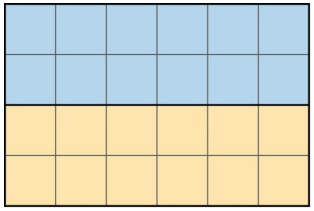
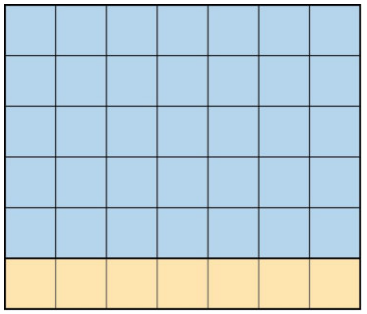
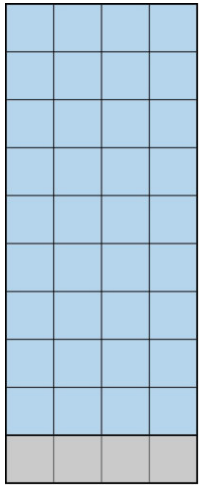
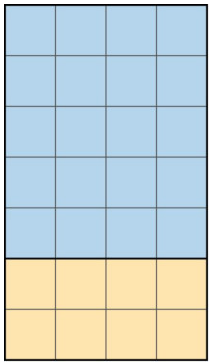
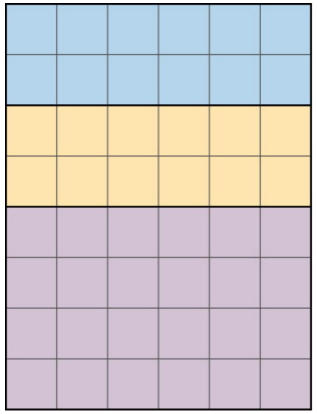
$(4 \times 2) \times 3$
8×3
24



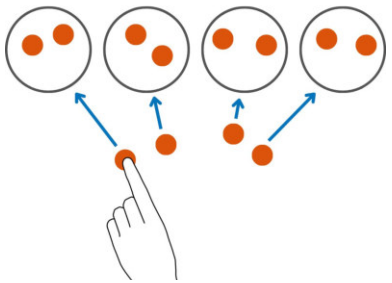
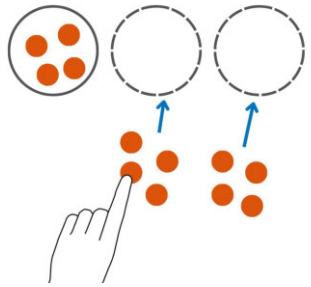
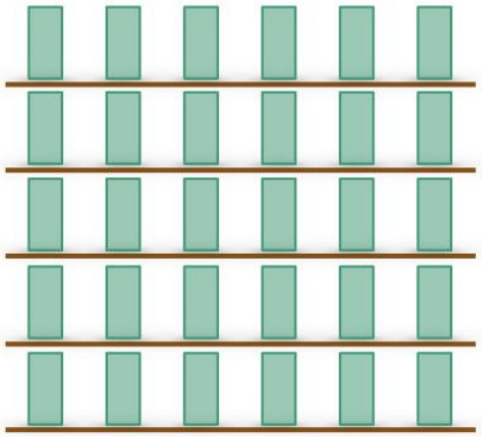
Developing Multiplication Fact Strategies

Fact fluency must develop **through use of models,** NOT through rote memorization. Students simultaneously explore properties of multiplication through composition and decomposition which build relational thinking strategies.

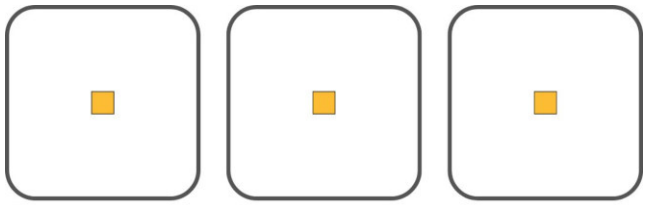
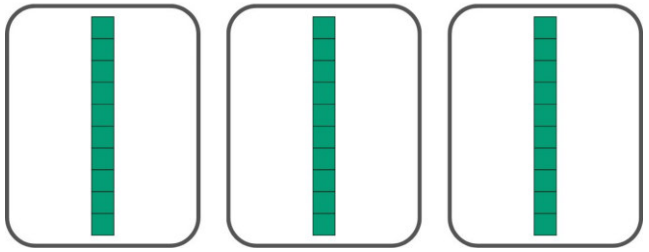
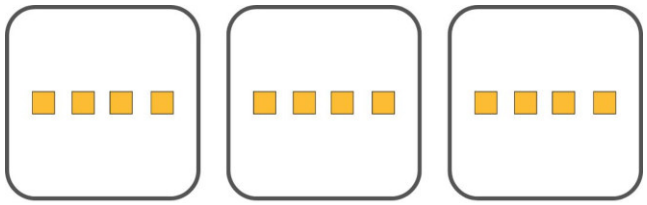
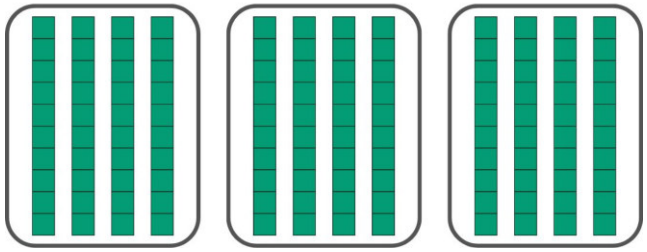
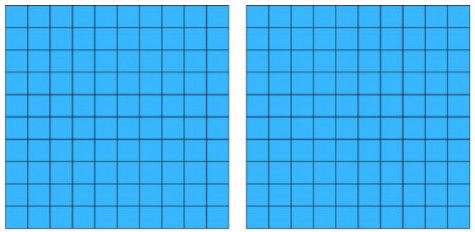
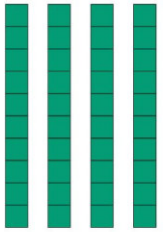
Below we show **examples of how students **might** derive multiplication facts. These examples are not meant to prescribe certain strategies that must be used.*

Practices and uses known facts including **0**, **1s**, **2s**, **5s** and **10s** facts (referred to as foundational facts)	Derives **3s** facts and **4s** facts using foundational facts, or other derived facts.	Derives **6s**, **9s** facts using foundational facts or other derived facts.	Derives **7s** and **8s** facts using foundational facts or other derived facts.
2 x 6 = 6 + 6  **Multiplying by 2** is like adding the same number twice (doubling). I can quickly **multiply by 5 or 10** by skip counting.  5, 10, 15, 20 **Multiplying by 10** is based on place value.  4 tens = 40 10, 20, 30, 40	3 x 6 = (2 x 6) + 6  **To multiply by 3,** I can just double the other factor and then add one more group. **To multiply by 4,** I can just double the other factor and then double it again.  4 x 6 = 2 x (2 x 6)	6 x 7 = (5 x 7) + 7  **To multiply by 6,** I can just multiply the other factor by 5 and then add one more group. **To multiply by 9,** I can just multiply the other factor by 10 and then subtract one group.  9 x 4 = (10 x 4) - 4	7 x 4 = (5 x 4) + (2 x 4) **To multiply by 7,** I can decompose into parts I know, like 5 and 2, and then add them together.  **To multiply by 8,** I can just double the other factor three times.  8 x 6 = 2 x 2 x (2 x 6)

Division - Composition and Decomposition *Students model both partitive and quotitive situations.*

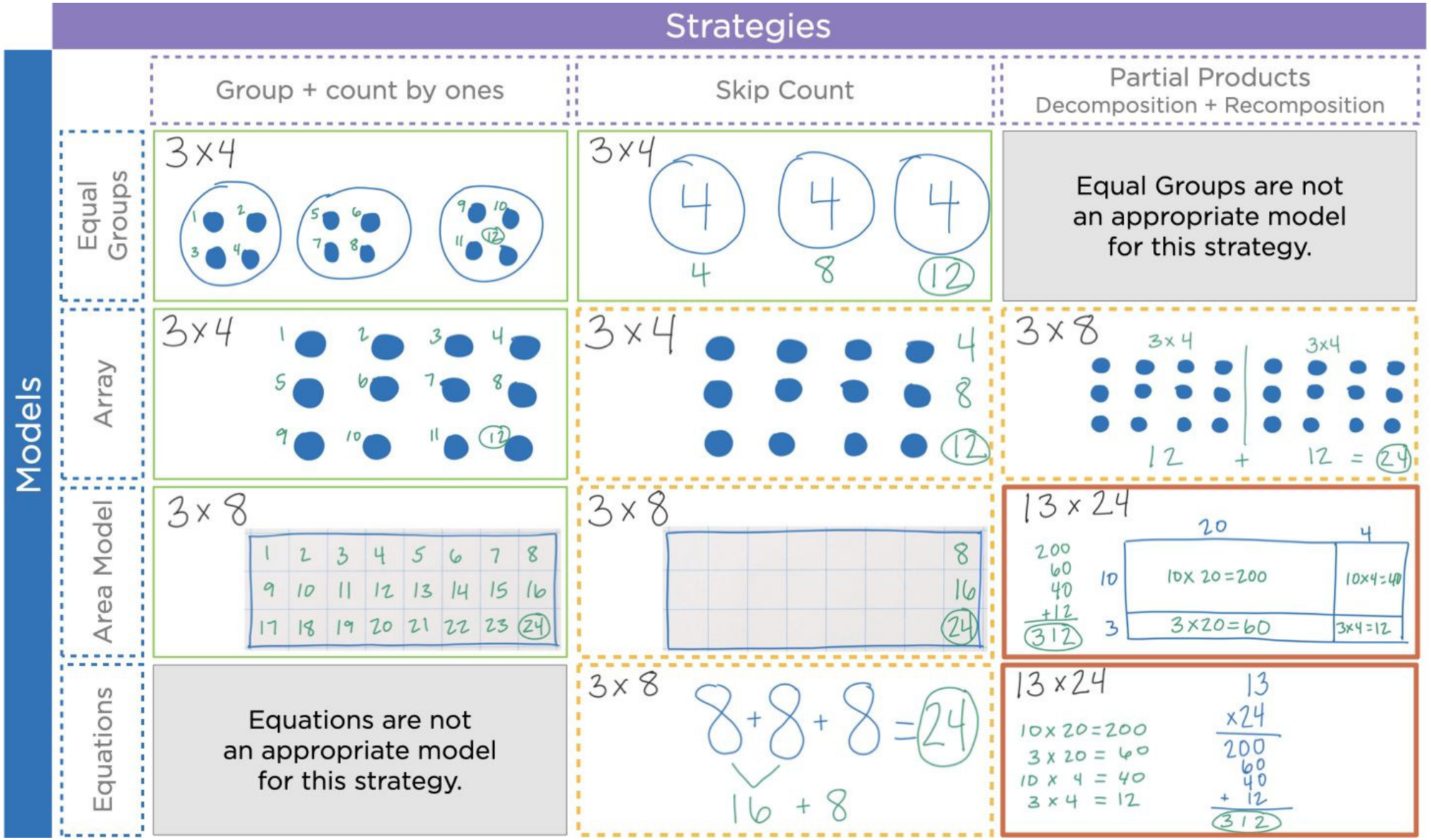
Shares by ones into equal sized subgroups- shares individual objects until all items are shared in groups equally (partitive division). Uses repeated subtraction of equal size groups or sharing in larger chunks (sharing by 2s, 3s, 4s and 5s) until all of the items have been removed from the total (quotitive division).	Organizes groups into rows or columns based on total number of objects and the given number of rows or columns. Students will start to use the missing factor as the answer.	Uses the area model to determine missing side length (missing dimension) through 10 x 10. Uses inverse relationship, and considers the missing factor problem for multiplication to solve a division problem.
Partitive I have 12 cookies to share equally across 4 plates. **How many cookies on each plate?** 12 (in all) ÷ 4 (groups) = # in each group  ***Quotative*** I have 12 cookies. Each plate holds 4 cookies. **How many plates do I have?** 12 (in all) ÷ 4 (in each group) = # of groups 	I have 30 books to organize on 5 shelves. **How many books are on each shelf?** 30 ÷ 5 = ___ 5 x ___ = 30 How many books per shelf? 5 rows (shelves) 	I want to build a garden that is 24 sq. feet. One side of the garden will be 4 feet long. **How long does the other side need to be?** ? 4 × 24 24 ÷ 4 = ___ ? 4 × 24 4 x ___ = 24

Composing and Decomposing Using Base Ten Units and Place Value - 1s, 10s, 100s *(Students must use models to build understanding along this trajectory. Models should support students developing understanding of the magnitude of digits in their place values.)*

Uses place value understanding to multiply single digit times 10. This involves extending understanding of ***single digit x single digit*** to ***single digit x a group of ten***	Uses place value understanding to multiply a single digit by multiple of 10. This involves extending understanding of ***single digit x single digit*** to ***single digit x multiple of ten***	Students decompose any number through expanded notation.
3 groups of 1 3 x 1 = 3  *so I know…* 3 groups of 10 3 x 1 ten = 3 tens = 30 3 x 10 = 30 	3 groups of 4 3 x 4 = 12  *so I know…* 3 groups of 40 3 x 4 tens = 12 tens = 120 3 x 40 = 120 	247 = (2 x 100) + (4 x 10) + (7 x 1)  2 x 100  4 x 10  7 x 1

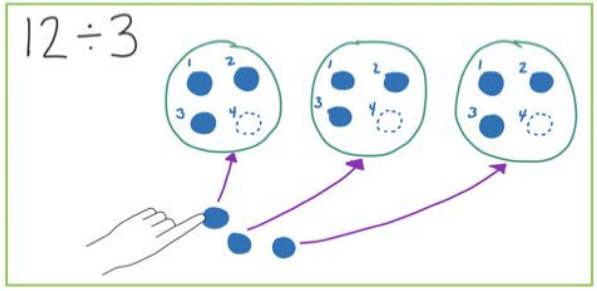
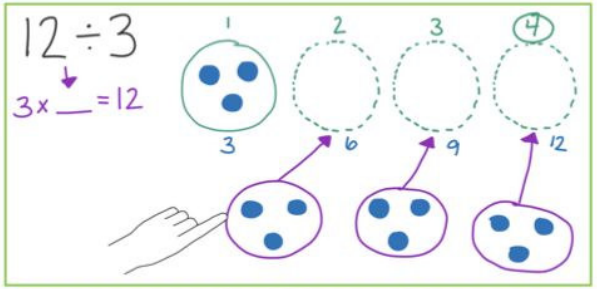
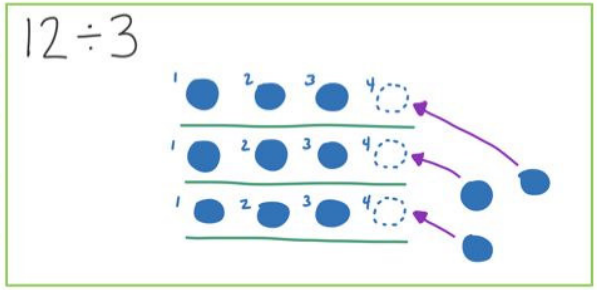
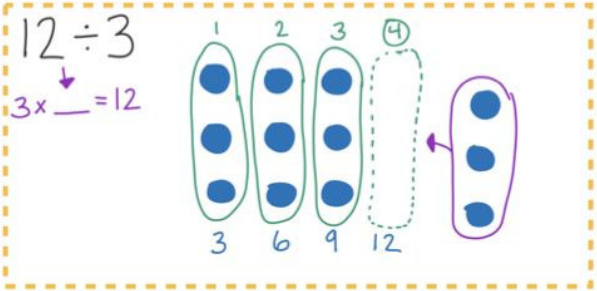
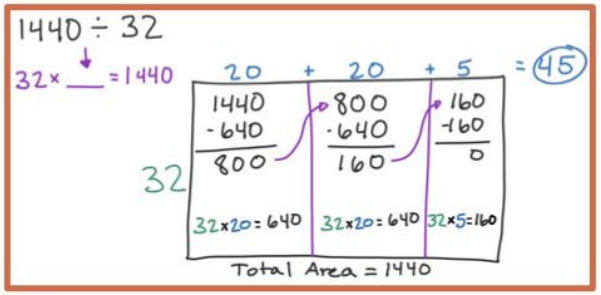
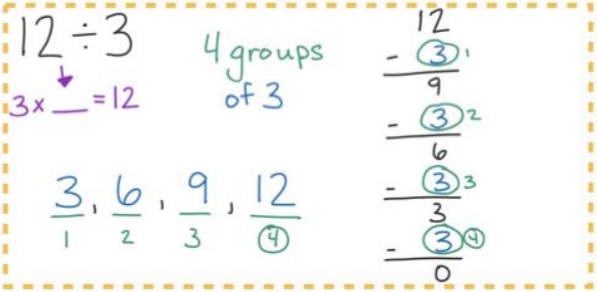
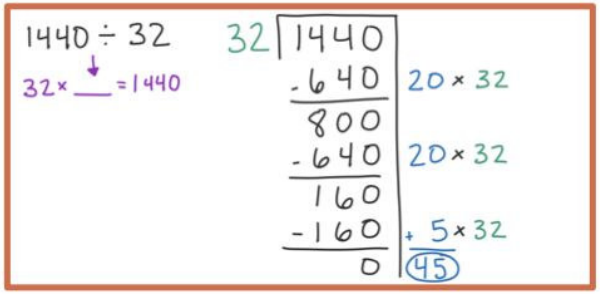
Models and Strategies for Multiplication (Across Grades 3-4)

Grade 3 | Grade 3 + 4 | Grade 4



Models and Strategies for Division (Across Grades 3-4)

Grade 3 | Grade 3 + 4 | Grade 4

Models	Strategies: Grouping/Fair Share	Strategies: Repeated Subtraction or Skip Counting	Strategies: Partial Quotients (Decomposition + Recomposition)
Equal Groups			Equal Groups are not an appropriate model for this strategy.
Array or Area Model			
Equations	Equations are not an appropriate model for this strategy.		

Multiplicative Reasoning

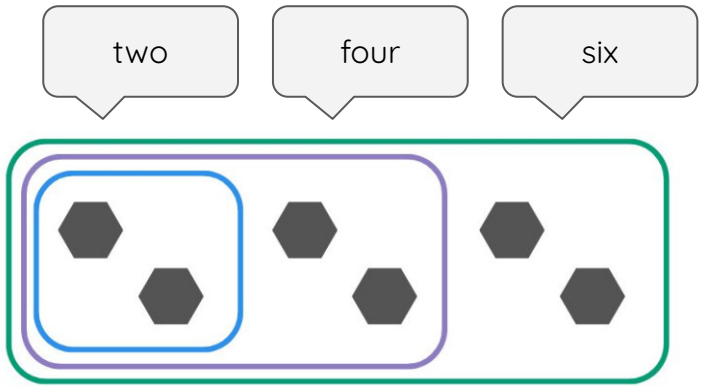
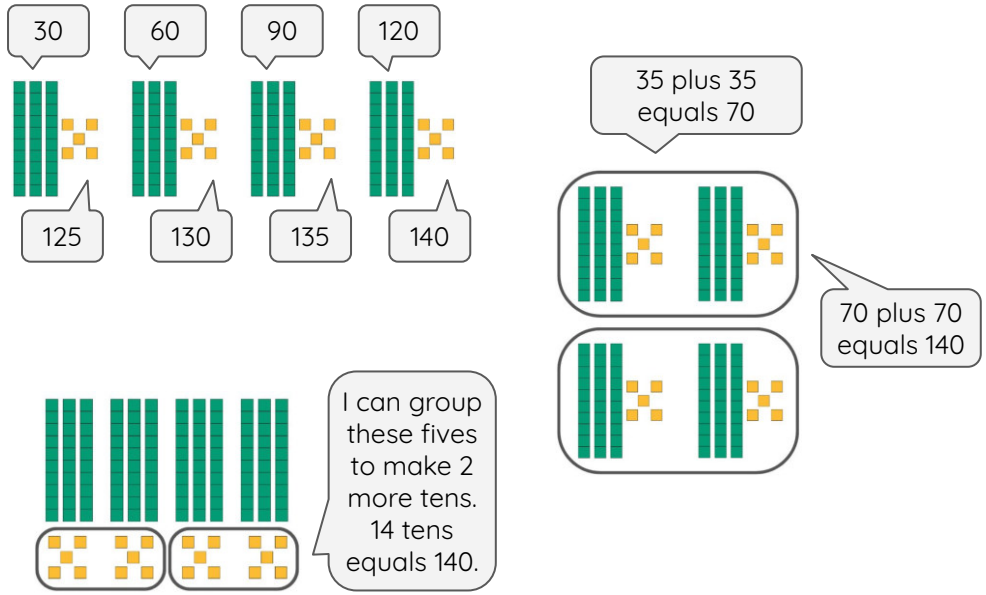
Grade Four HLC

Multiply and divide within 1000 within context and with equations.

September → **Grade Four Learning Progressions** → June

Students must use models to build understanding along this trajectory and interact with a variety of contexts for multiplication and division. Models will continue to support students' ability to unitize–understand a group or collection of items represents "one." (For example, one group of 5 consists of 5 individual items but is classified as one group.)

Counting by Equal Groups (Unitizing) to Extend Multiplicative Understanding

Skip counts the equal sized groups or uses repeated addition to tell the cumulative total of each group.	Combines equal sized groups in flexible ways to begin to explore partial products.
	

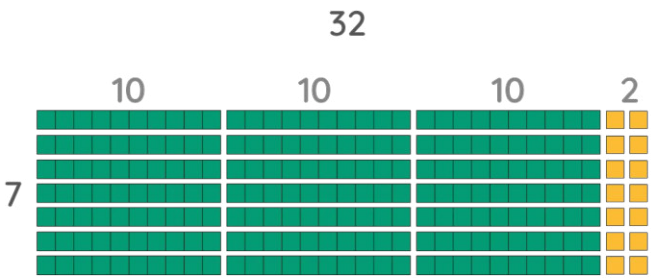
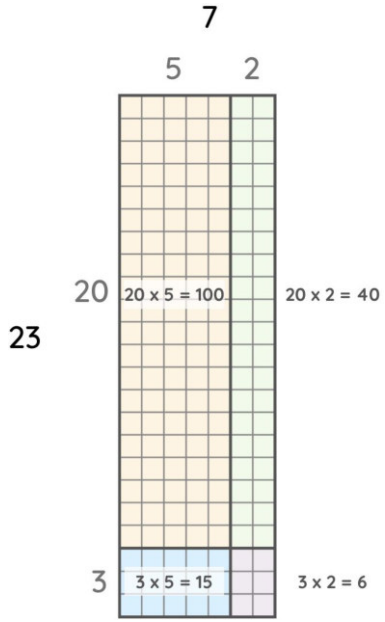
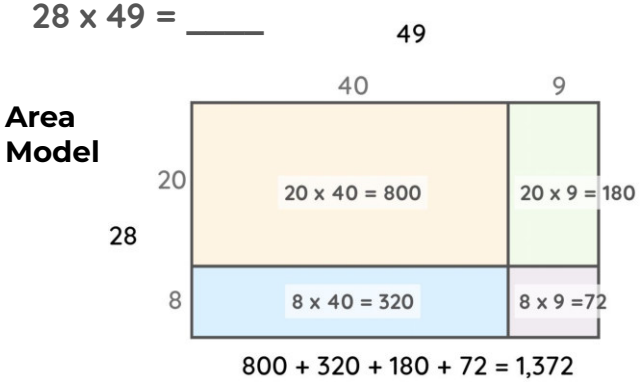
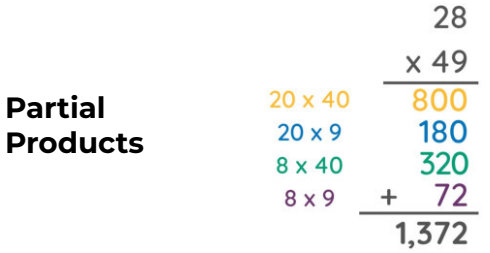
Operations: Multiplication and Division

Students must use models to build understanding along this trajectory and interact with a variety of contexts for multiplication and division. Models should support students developing understanding of the magnitude of digits in their place values. In Grades 3 and 4, place value understanding is multiplicative: 245 = 2(100) + 4(10) + 5(1) Students also use relational thinking when composing, decomposing and recomposing.

***Students are maintaining and using their fact strategies to solve basic facts through 100 within context and with equations.*

Multiplication - Composition and Decomposition

Students derive strategies through the use of area models, decomposition of numbers, and relational thinking with known facts.

Uses the area model or array to decompose into smaller arrays.	Uses the area model for products to 1000 (100 x 100) to understand length and width as the dimensions of a 1 x 1 square unit. Students may use a variety of strategies to calculate the product.	Decomposes the side lengths to use the distributive property with numbers through 100 x 100. Uses the partial products method to solve multiplication problems with numbers through 100 x 100
$7 \times 32 = 7 \times (10 + 10 + 10 + 2)$ $7 \times 32 = (7 \times 10) + (7 \times 10) + (7 \times 10) + (7 \times 2)$ 	$23 \times 7 = (20 + 3) \times (5 + 2)$ $23 \times 7 = (20 \times 5) + (20 \times 2) + (3 \times 5) + (3 \times 2)$ 	$28 \times 49 =$ _____  

Properties of Multiplication *(These properties are investigated throughout the year with different numbers and problem situations. The sequence of how the properties appear below does not suggest the order in which to explore them. Many times the properties can be explored simultaneously with student work.)*

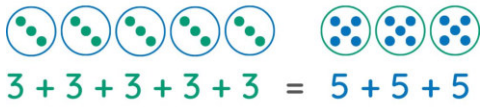
Identity Property

$7 \times 1 = 7$

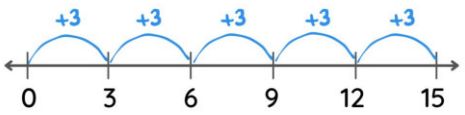
I have seven groups of one, which is the same as seven.

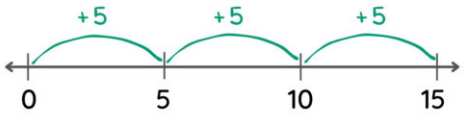
Commutative Property

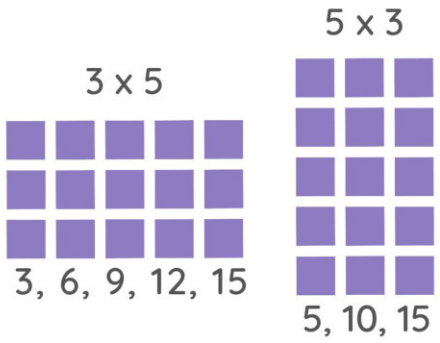
$3 \times 5 = 5 \times 3$



Three groups of five is the same value as five groups of three.





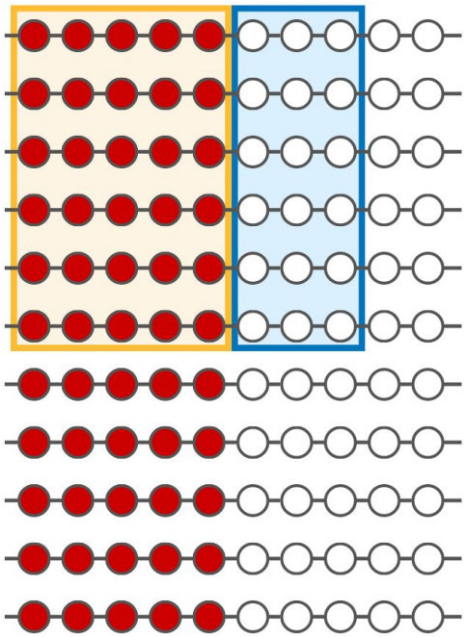


Distributive Property

$6 \times 8 = (6 \times 5) + (6 \times 3)$

Eight groups of six is the same value as six groups of five plus six groups of three.

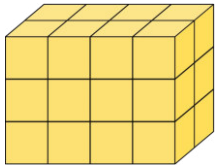
$30 + 18 = 48$



100 bead rack illuminates benchmark numbers

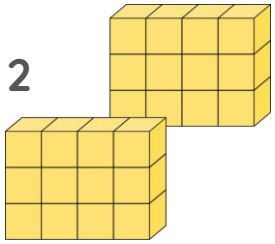
Associative Property

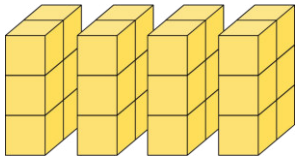
$4 \times 3 \times 2$



I can multiply the dimensions in any order to find the volume.

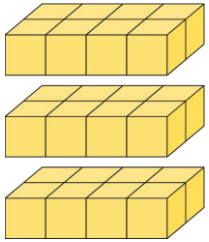
$(4 \times 3) \times 2$
12×2
24





$4 \times (3 \times 2)$
4×6
24

$(4 \times 2) \times 3$
8×3
24



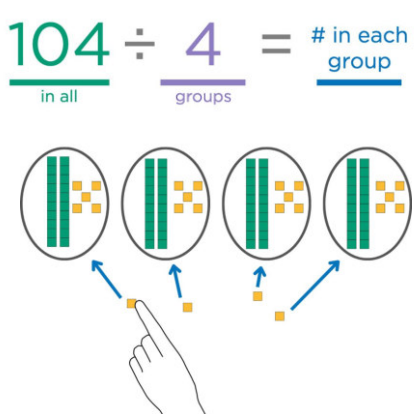
Division - Composition and Decomposition *(Students model both partitive and quotitive situations)*

Shares equal sized portions from the total to each group using benchmark sized quantities (10, 5, 2 and then 1s) from a whole within 1,000. AND/OR Subtracts equal sized groups of the divisor from the total.	Uses inverse relationship, and considers the missing factor problem for multiplication to solve a division problem. May use partial products and foundational facts to build up to the total.	Uses partial quotients, removes larger-sized products using the divisor as a factor, multiples of benchmark numbers, and multiplication facts.

Partitive

I have 104 cookies to share equally across 4 platters.

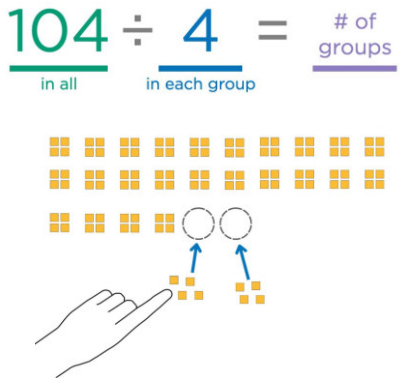
How many cookies for each platter?



Quotitive

I have 104 cookies. I need to put four cookies in each bag.

How many bags can I make?



480 ÷ 15 = ____

15 x ____ = 480

?

15	480

1 x 15 = 15

2 x 15 = 30

10 x 15 = 150

20 x 15 = 300 → 480

32 x 15 = 480

(2 x 15) + (10 x 15) + (20 x 15) = 480

1,440 ÷ 32 = ____

32 x ____ = 1,440

?

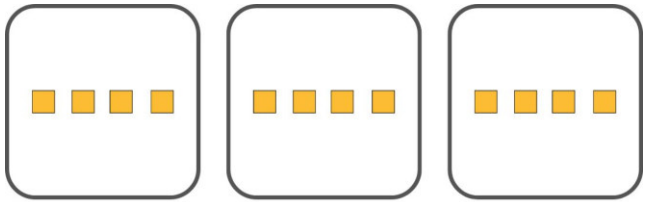
32	20	+	20	+	5
	1,440		800		160
	- 640		- 640		- 160
	800		160		0
	1,440				

32	1,440	
	- 640	20 x 32
	800	
	- 640	20 x 32
	160	
	- 160	+ 5 x 32
	0	45

Composing and Decomposing Using Base Ten Units and Place Value - 1s, 10s, 100s, 1000s *(Students must use models to build understanding along this trajectory. Models should support students developing understanding of the magnitude of digits in their place values.)*

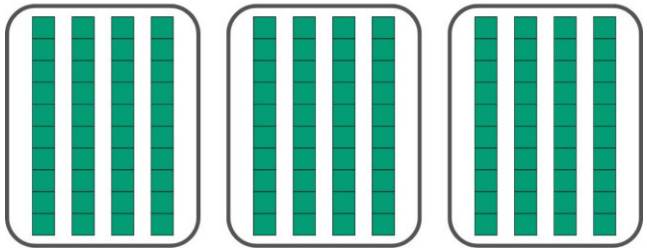
Uses place value understanding to multiply a single digit by multiple of 10. This involves extending understanding of ***single digit*** **x** ***single digit*** to ***single digit*** **x** ***multiple of ten***	Understands expanded notation of numbers to 1000. Students are thinking multiplicatively about place value. Understands that each place is 10 times more as you move to the left.	Uses place value understanding to decompose factors to multiply using area model and partial products.

3 groups of 4
3 x 4 = 12



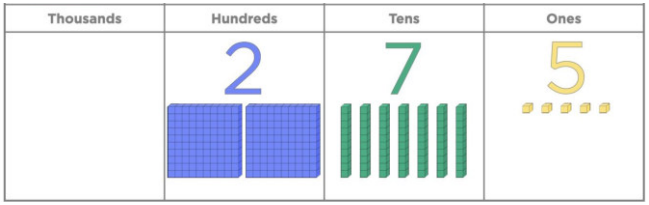
so I know…

3 groups of 40
3 x 4 tens = 12 tens = 120
3 x 40 = 120



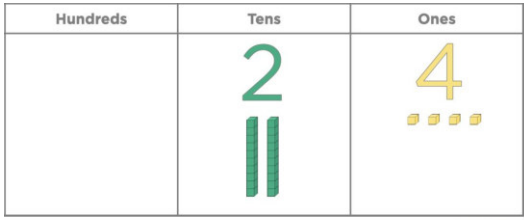
Expanded Notation

275 = (2 x 100) + (7 x 10) + (5 x 1)

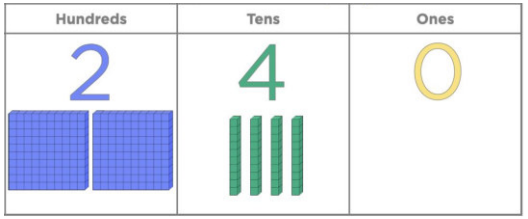


Multiplicative View of Place Value

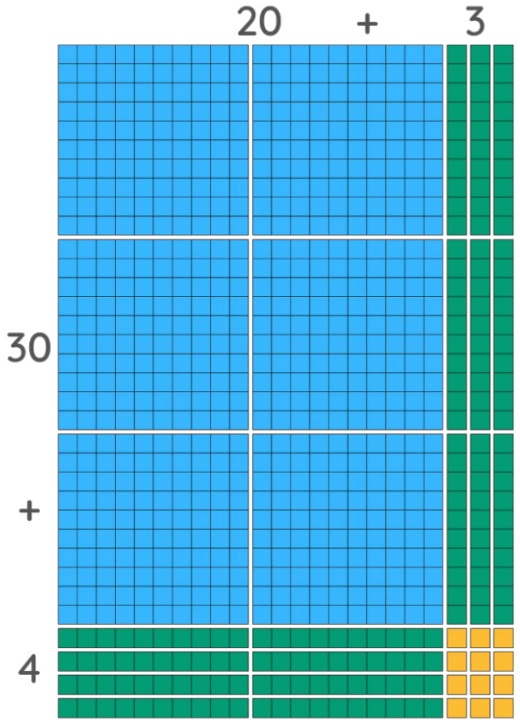
Each place is 10 x more - NOT "adding a zero" or "moving the decimal point."



24 x 10 = ___

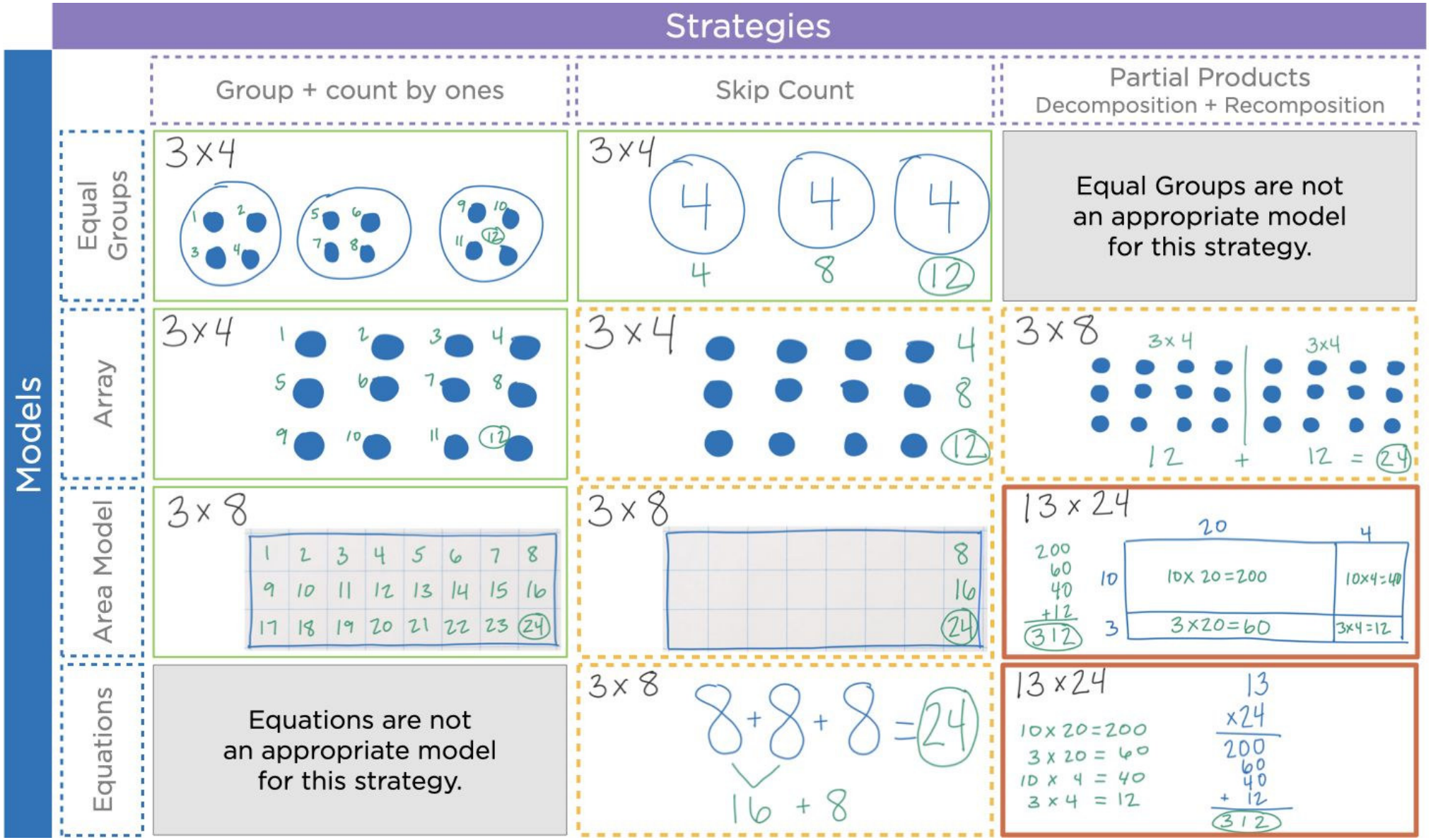


23 x 34 = (20 + 3) x (30 + 4)
20 x 30 = 600
30 x 3 = 90
4 x 20 = 80
4 x 3 = 12
600 + 90 + 80 + 12 = 782



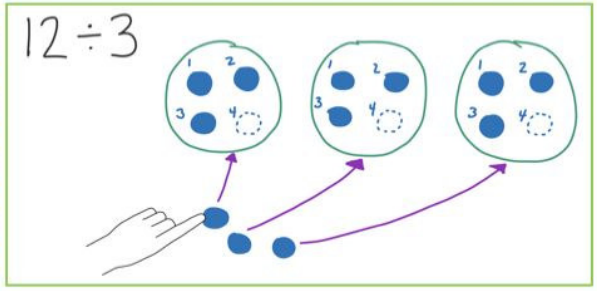
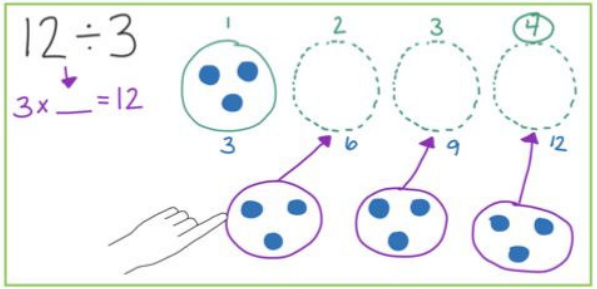
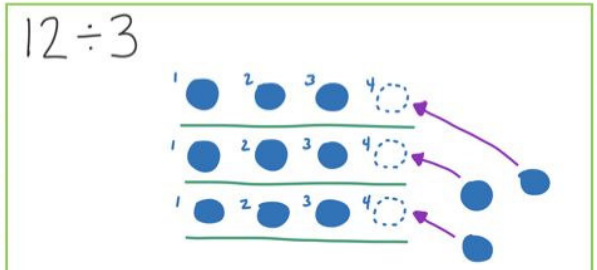
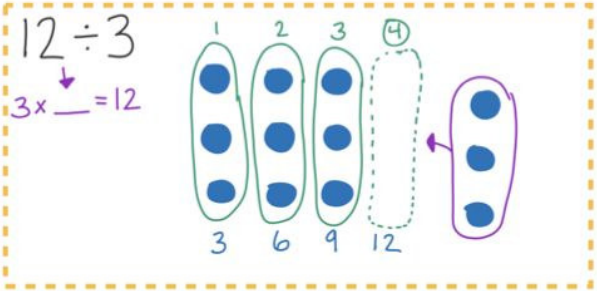
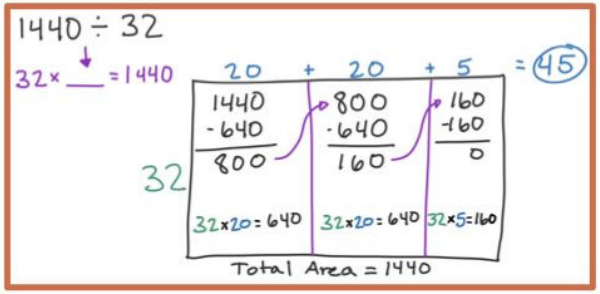
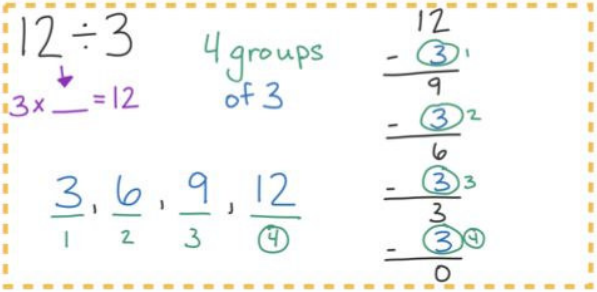
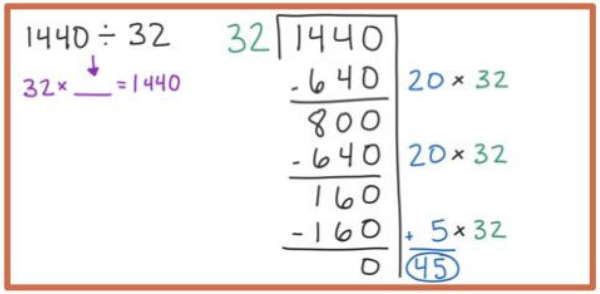
Models and Strategies for Multiplication (Across Grades 3-4)

Grade 3 | Grade 3 + 4 | Grade 4



Models and Strategies for Division (Across Grades 3-4)

Grade 3 | Grade 3 + 4 | Grade 4

Models	Strategies: Grouping/Fair Share	Strategies: Repeated Subtraction or Skip Counting	Strategies: Partial Quotients (Decomposition + Recomposition)
Equal Groups			Equal Groups are not an appropriate model for this strategy.
Array or Area Model			
Equations	Equations are not an appropriate model for this strategy.		

Fractions

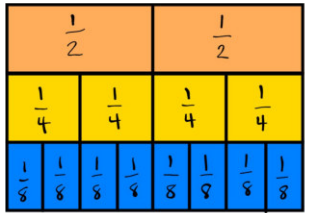
Foundational Understanding of Fractions

The 5th grade HLC progression focuses on operating with fractions. Prior to operating with fractions, students should have opportunities to compare and order fractions, reason about the relative size of fractions and develop understanding about equivalent fractions.
(see 5th grade HLC progression on the subsequent pages)

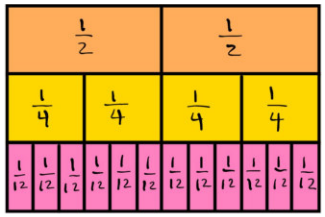
Equipartitioning

Equipartitioning is directly related to multiplication factors.

Example 1: Folding paper to make eighths, first fold the whole in half. Then fold each of those pieces in half. Then fold each of those in half to have eight equal parts. This connects to 8=2 x 2 x 2 as we break apart the whole in half, in half again, and again once more.



Example 2: In making twelfths, first fold the whole in half. Then fold each of those pieces in half. Then fold each of those in thirds to have twelve equal parts. This connects to 12=2 x 2 x 3 as we break apart the whole in half, in half again, and then in thirds.

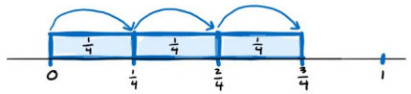


Visual Representations

Students need to interact with multiple visual representations of fractions.

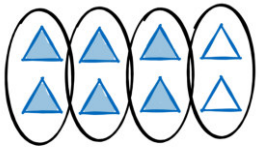
Tape Diagram / Number Line

$\frac{3}{4}$



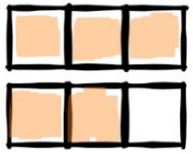
Set Model

$\frac{3}{4}$



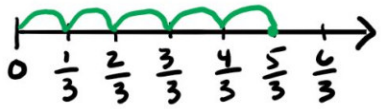
Linear Model

$\frac{5}{3}$



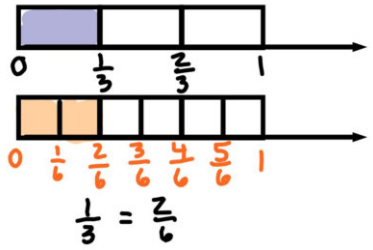
Number Line

$\frac{5}{3}$

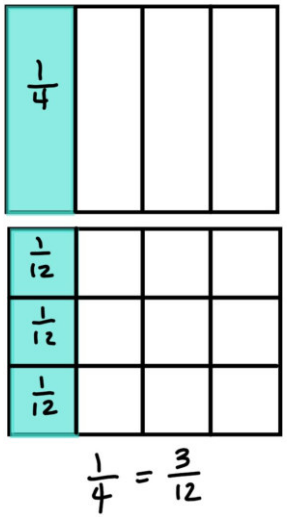


Equivalence using Visual Representations

Students need to explore equivalence through use of many different models.



$\frac{1}{3} = \frac{2}{6}$

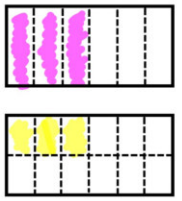


$\frac{1}{4} = \frac{3}{12}$

Comparing and Ordering

There are a variety of reasoning strategies to compare/order fractions. Below are a few common strategies.

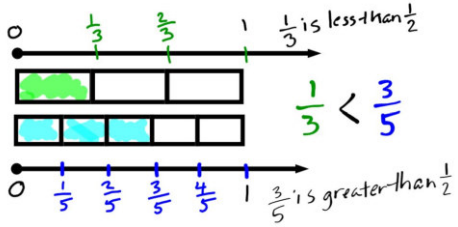
Common Numerator



$\frac{3}{6} > \frac{3}{12}$

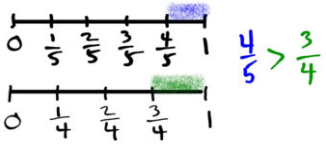
$\frac{1}{6}$ is larger than $\frac{1}{12}$

Comparison to 0, ½, 1... (< or > ½)



$\frac{1}{3} < \frac{3}{5}$

Distance from a benchmark



$\frac{4}{5} > \frac{3}{4}$

$\frac{4}{5}$ is closer to 1 than $\frac{3}{4}$

Grade Five HLC

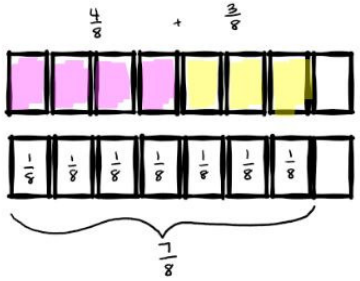
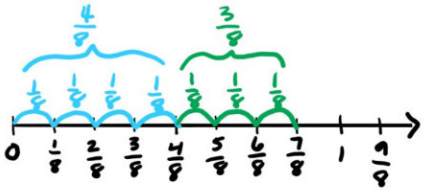
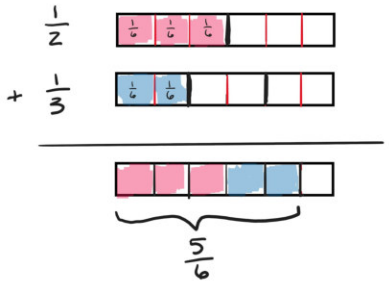
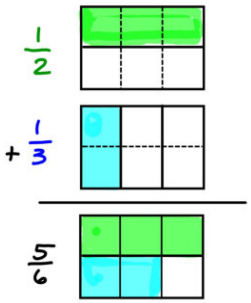
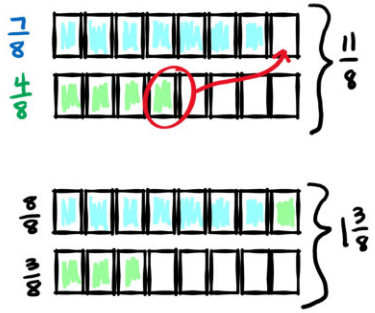
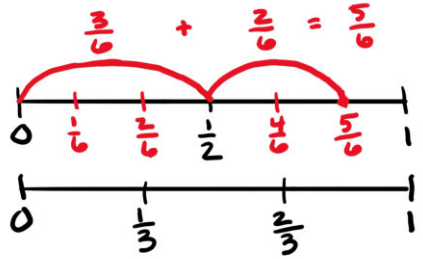
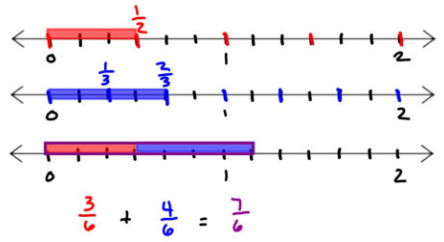
Add, subtract, multiply and divide with fractions (in context and in equations) using visual representations

September → **Grade Five Learning Progressions** → June

Students must use models to build understanding along this trajectory and interact with a variety of contexts of adding, subtracting, multiplying and dividing fractions. **NO algorithms before conceptual understanding is SOLID. Introducing algorithms too early interrupts and/or has a negative impact on understanding**

Adding & Subtracting Fractions

Students move from adding/subtracting with same denominators to adding/subtracting with different denominators.

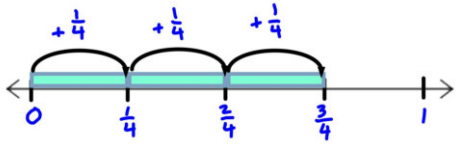
Compose and decompose using unit fraction knowledge	Add unit fractions using the area model. Partition models into the same number of equal parts	Use area models in relation to a benchmark number	Add fractions using double number lines
Students compose $\frac{7}{8}$ by adding $\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}$ has the same value as $\frac{4}{8}+\frac{3}{8}$ **Linear Model**  **Jumps on a Number Line** 	$\frac{1}{2}+\frac{1}{3}$ has the same value as $\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$  	"$\frac{7}{8}$ *is* $\frac{1}{8}$ *away from 1 whole.*" $\frac{7}{8}+\frac{1}{2}=\frac{7}{8}+\frac{4}{8}=1\frac{3}{8}$ 	$\frac{1}{2}+\frac{1}{3}=\frac{5}{6}$  $\frac{1}{2}+\frac{2}{3}=\frac{7}{6}$ 

Multiplying Fractions

Students will interact with a whole number times unit fractions, then a whole number times a fraction less than 1, then move to unit fractions times unit fractions and finally to all other fraction multiplication types. Students will recognize and discover the patterns that lead to the standard algorithm.

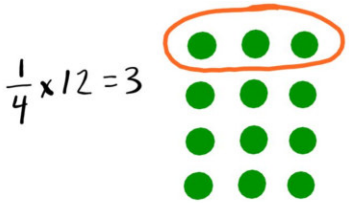
Use visual representations to multiply a unit fraction and a whole number	Use area models to multiply unit fractions like ⅓ x ½	Use area models to multiply non-unit fractions like ¾ x ½	Use area models to multiply any two fractions.

Tape Diagram/Number Line

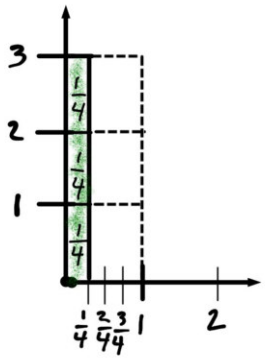


$3 \times \frac{1}{4} = 3 \text{ groups of } \frac{1}{4} = \frac{3}{4}$

Set Model

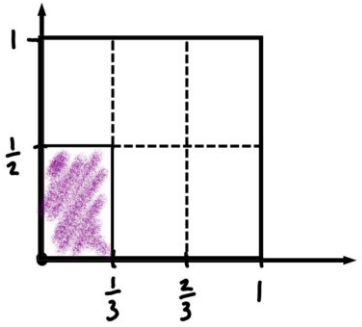


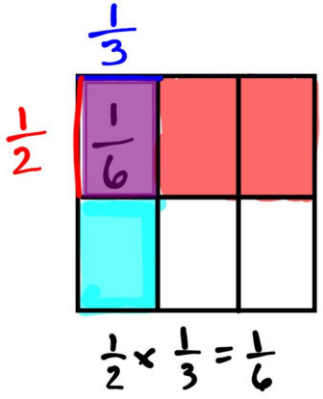
Area Model



" What is $\frac{1}{3}$ of $\frac{1}{2}$?"

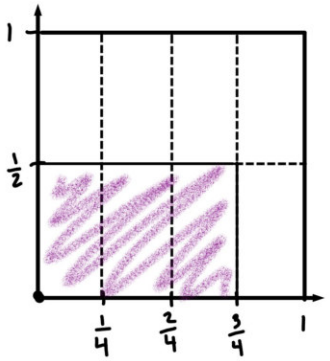
$\frac{1}{2} \times \frac{1}{3}$

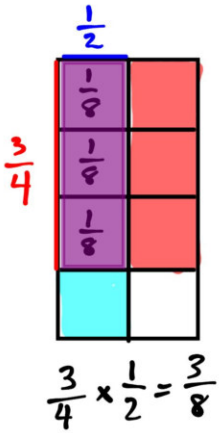




" What is $\frac{3}{4}$ of $\frac{1}{2}$?"

$\frac{3}{4} \times \frac{1}{2}$

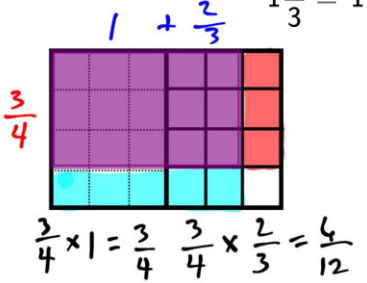




$1\frac{2}{3} \times \frac{3}{4}$

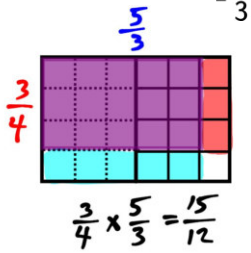
Expanded Form

$1\frac{2}{3} = 1 + \frac{2}{3}$

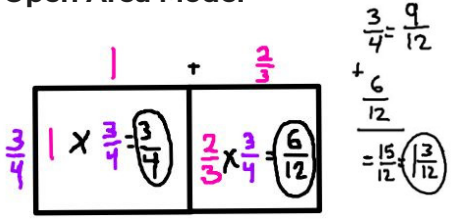


Fractions > 1

$1\frac{2}{3} = \frac{5}{3}$

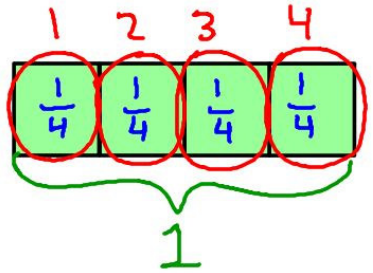
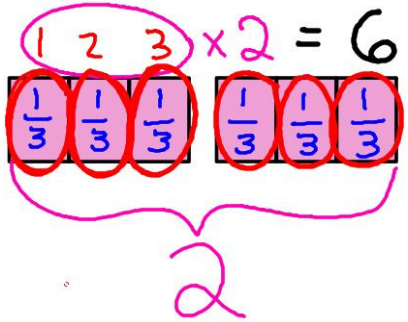
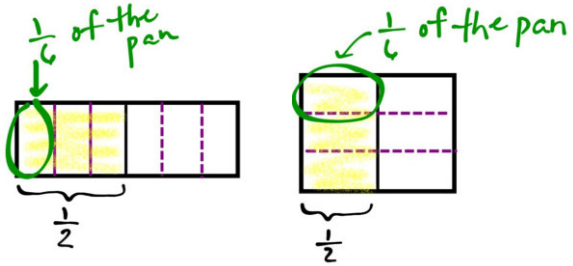
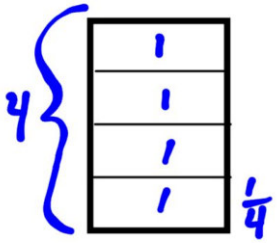
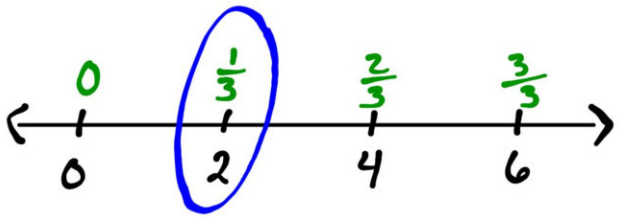
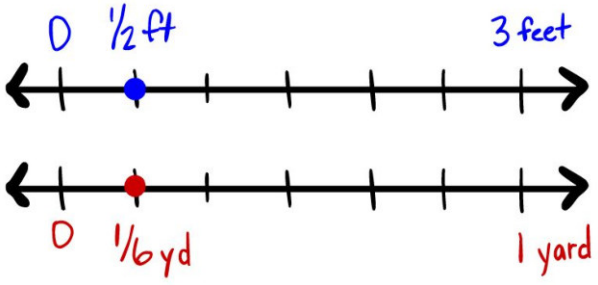


Open Area Model



Dividing Fractions

In 5th grade, fraction division focuses only on problems with a whole number and a unit fraction (unit fractions by whole numbers, whole numbers by unit fractions). Students will be exposed to all problem types; partitive, quotitive, multiplicative change, measurement conversion and rectangular area.

Use visual representations to divide 1 by a unit fraction	Use visual representations to divide a whole number by a unit fraction	Use visual representations to divide a unit fraction by a whole number
$1 \div \frac{1}{4}$	$2 \div \frac{1}{3}$	$\frac{1}{2} \div 3$
Quotitive "How many groups of ¼ are there in 1?" "How many ¼ cup scoops of flour are there in 1 cup of flour?"	**Quotitive** "How many groups of ⅓ are there in 2?" "How many ⅓ foot bracelets can I make out of 2 feet of ribbon?"	**Partitive** "There is ½ of a pan of brownies to be shared equally between 3 people. Each person will get ⅙ of a pan of brownies."
		
Multiplicative Change modeled with Rectangular Area "1 is ¼ of the total. Find the total." "1 gallon fills ¼ of the gas tank. How many gallons does the whole tank hold?"	**Multiplicative Change** "2 is ⅓ of the total. Find the total." "2 feet of rope is ⅓ of the total length. How long is the rope?	**Measurement Conversion** "I have ½ foot of ribbon. How many yards of ribbon do I have?"
		

Proportional Reasoning - Ratios

Grade Six HLC

Use visual representations to compare ratios, and solve problems including those involving unit rates and percentages

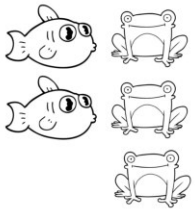
September → **Grade Six Learning Progressions** → June

Students must use visual representations to build understanding along this trajectory and interact with a variety of rates and ratios.
Be VERY cautious of introducing algorithms before conceptual understanding is SOLID

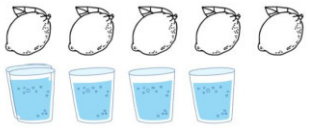
Critical Strategies: Look for and identify multiplicative relationships in tables and diagrams.

Visually represent and interpret a ratio between two quantities *(two quantities could connect part to part or part to whole)*	Find equivalent ratios *(Initially ratio tables are created through skip counting, then students recognize multiplicative relationships)*	Use multiplicative reasoning, unit rates and/or equivalency to compare ratios and solve more complex problems.

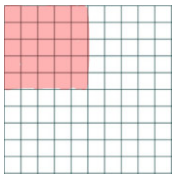
2 fish : 3 frogs



Mix A - Lemonade

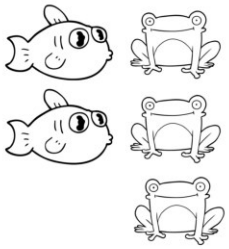


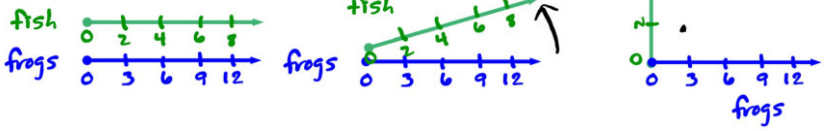
100 grid
25% of 100



Visual rep with iteration

Fish	Frogs
0	0
2	3
4	6
6	9
8	12

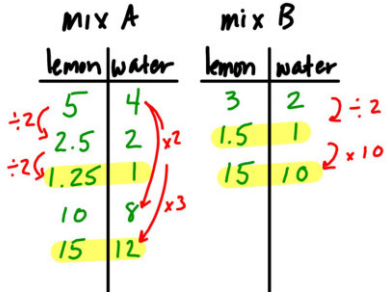




Cont. on next page

Ratio Table
Mix A: 5 cups of lemon to 4 cups of water
Mix B: 3 cups of lemon to 2 cups of water

Mix B is more lemony - it has more lemon to 1 part water or more water to 15 parts lemon.



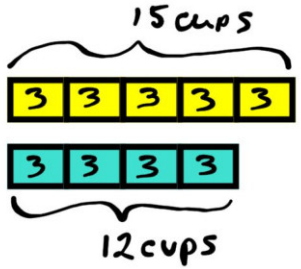
Cont. on next page

Find equivalent ratios **(cont.)** *(Initially ratio tables are created through skip counting, then students recognize multiplicative relationships)*	Use multiplicative reasoning, unit rates and/or equivalency to compare ratios and solve more complex problems. **(cont.)**

Multiplicative reasoning and scaling

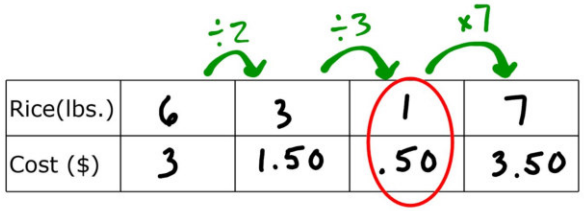
$$\frac{6\ fish}{9\ frogs} = \frac{2\ fish}{3\ frogs} = \frac{8\ fish}{12\ frogs}$$

If there are 15 cups of lemon, how much water is needed?



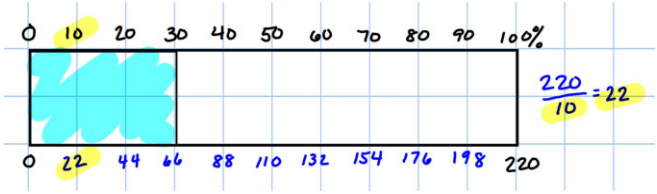
Identify unit rates in multiple contexts

6 pounds of rice costs $3. How much will it cost for 7 pounds of rice?



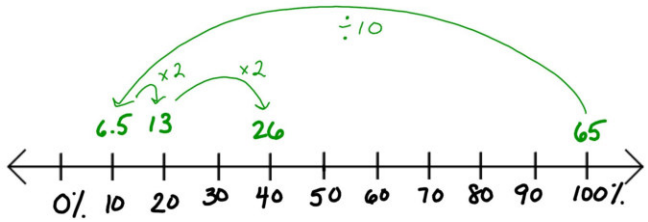
Percent Bar

66 is what percent of 220?



Double Number Line

40% of 65



Proportional Reasoning

Grade Seven Proportional Reasoning HLC

Recognize proportional relationships and identify the unit rate in tables, graphs, equations and in context.

September → **Grade Seven (PR) Learning Progressions** → June

Students must use visual representations to build understanding along this trajectory and interact with a variety of proportional contexts.
Be VERY cautious of introducing algorithms before conceptual understanding is SOLID

Critical Strategies: Look for and identify multiplicative relationships in tables and diagrams.
(The cross products algorithm does not qualify as demonstrating understanding.)

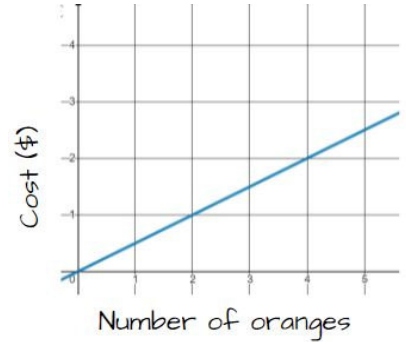
Look for and identify proportional relationships.	Determine the constant of proportionality (k)	Write an equation in terms of k

Visual Representations

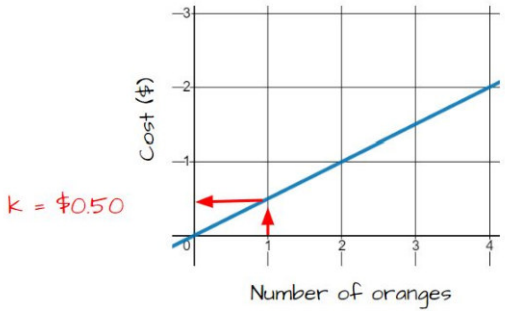
Tables

oranges	0	3	4	6
Cost ($)	0	1.50	2.00	3.00

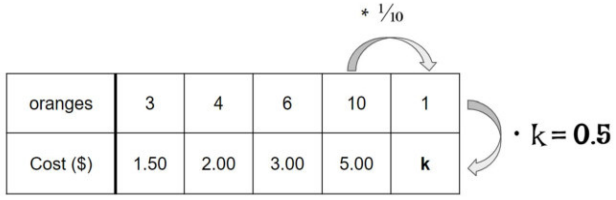
Graphs



Graphs



Tables

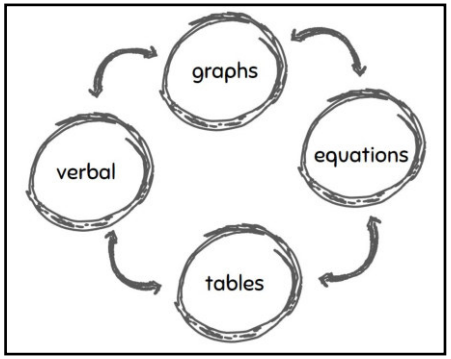


Equations

$$k = \frac{y}{x}$$

Can create y = **k**•x equation and move fluidly between graphs, tables and equations

y = cost in dollars
x = number of oranges
y = 0.5 x



Expressions and Equations

Grade Seven Expressions and Equations HLC

Operate with signed numbers and create equivalent expressions.

September → **Grade Seven (EE) Learning Progressions** → June

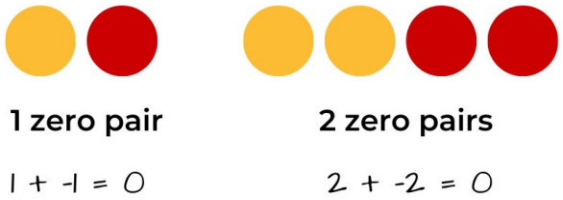
Students must use visual representations to build understanding along this learning progression. Algebra tiles are strongly recommended since students use tiles to model in elementary through high school mathematics. **Be VERY cautious of introducing algorithms before conceptual understanding is SOLID**

Critical Strategies: Zero pairs are useful tools when working with signed numbers.

Understanding Integers
Possible contexts: temperature, money/debt, elevation

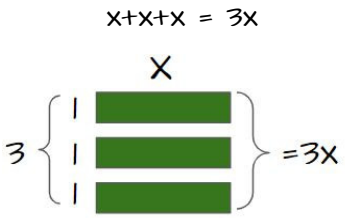
Models:
- Number line
  (horizontal + vertical)
- 2 sided chips/Algebra tiles

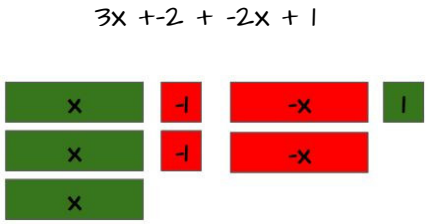
Students should:
- Compare integers by thinking about their distance from zero and using >, <, =
- Use zero pairs



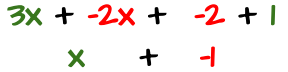
Build and Create Equivalent Expressions
Possible contexts: temperature, money/debt, elevation, area, perimeter, emotions scale

Algebra Tiles

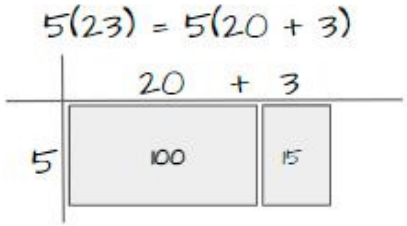


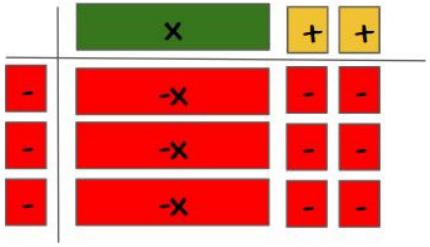


Collect Like Terms



Distributive Property
Make connections between **whole number** and **variable models**



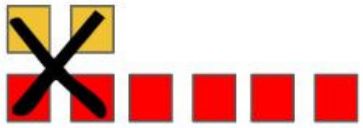


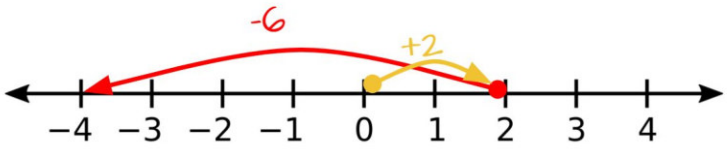
Cont. on next page

Operations with Integers

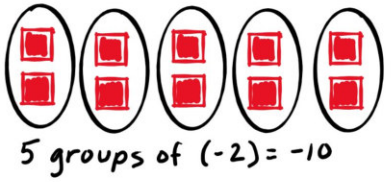
Possible contexts: temperature, money/debt, elevation, area, perimeter, emotions scale

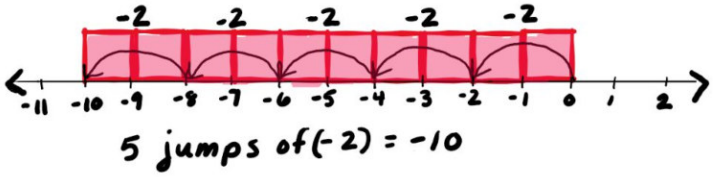
Add 2 + (-6) = -4





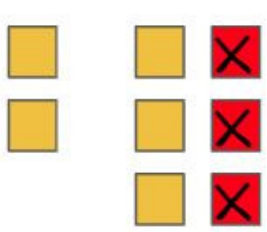
Multiply 5 x (-2) = -10



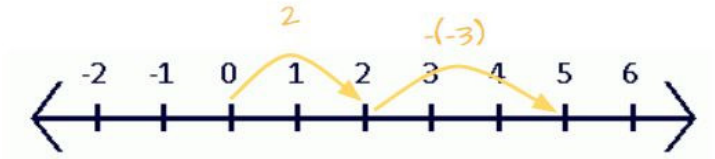


Notice Patterns:
-3 x 3 = -9
-3 x 2 = -6
-3 x 1 = -3
-3 x 0 = 0
-3 x -1 = 3
-3 x -2 = 6
etc...

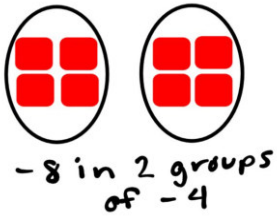
Subtract 2 - (-3) = 5

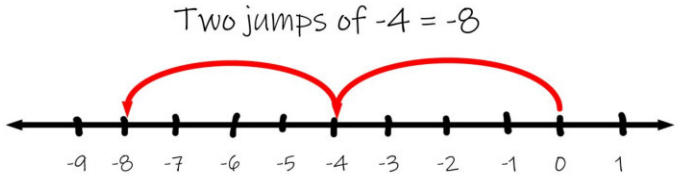


By adding 3 "zero pairs" you can then take away 3 negatives.



Divide -8 ÷ 2 = -4





ALL LEARNERS NETWORK

Grade Eight HLC Learning Progressions - DRAFT

(Expressions and Equations)

Expressions and Equations

Grade Eight Expressions and Equations HLC

Solve equations for unknowns which may include signed numbers.

September → **Grade Eight (EE) Learning Progressions** → June

Students must use visual representations to build understanding along this learning progression. Algebra tiles are strongly recommended since students use tiles to model in elementary through high school mathematics. **Be VERY cautious of introducing algorithms before conceptual understanding is SOLID**

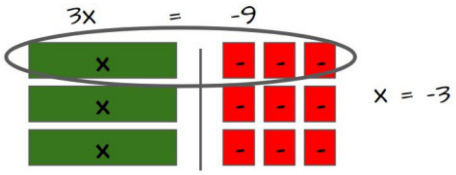
Critical Strategies: Use inverse operations for solving problems

Solving Equations

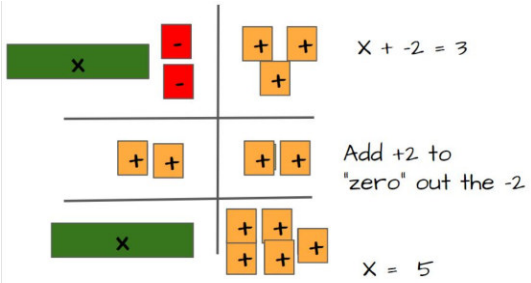
Connecting visual representations to algebraic notation

Algebra Tiles

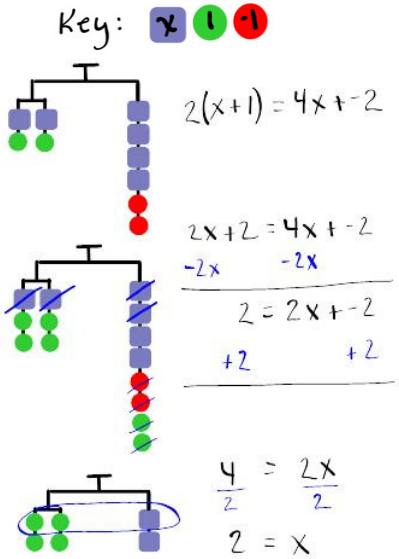
Grouping/ fair share



Add tiles to make zero pairs



Hangar Diagram



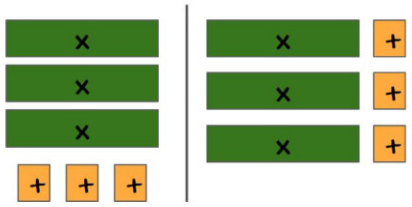
Other representations: Money bags, pan balance

Understanding the number and meaning of solution(s)

The solution is the value(s) of x is that makes a given equation true

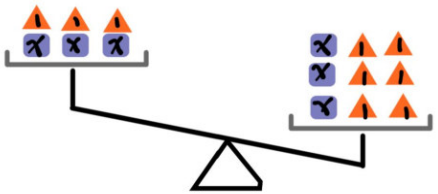
In context of solving an equation:

x = infinitely many solutions:



In this example, 3x + 3= 3(x+1) shows that x can have any value, so x has infinitely many solutions.

x has no solutions:



In this example, 3x + 3 = 3(x + 2), there is no value of x that will make this true, so x has no solution.

ALL LEARNERS NETWORK

Grade Eight HLC Learning Progressions - DRAFT
(Linear Relationships)

Linear Relationships

Grade Eight Linear Relationships HLC

Understand linear relationships using contexts, tables, graphs and equations. Make connections among representations of linear relationships.

September → **Grade Eight (LR) Learning Progressions** → June

Students must use visual representations to build understanding along this trajectory and interact with a variety of linear contexts.
Be VERY cautious of introducing algorithms before conceptual understanding is SOLID

Critical Strategies: Finding the rate of change between two quantities (x and y) and the vertical intercept or initial value

Verbal (in context)

At Monster Ski Mountain, the cost for a Bash Badge is $100. Once you purchase a badge, you then pay $20 for each day you ski.

Initial Value (Starting cost/out of pocket) = $100

Rate of Change= $20 for every ticket you purchase.
(an increase of $20 for every 1 ticket)

***Note:** the vocabulary **"initial value"** & **"rate of change"** comes directly from Common Core*

Tables

Initial Value of **100**
(when x is 0, y is 100)

# of tickets (x)	Total Cost in dollars (y)
0	100
1	120
2	140
5	200
12	240

Δx=+4 Δy=+80

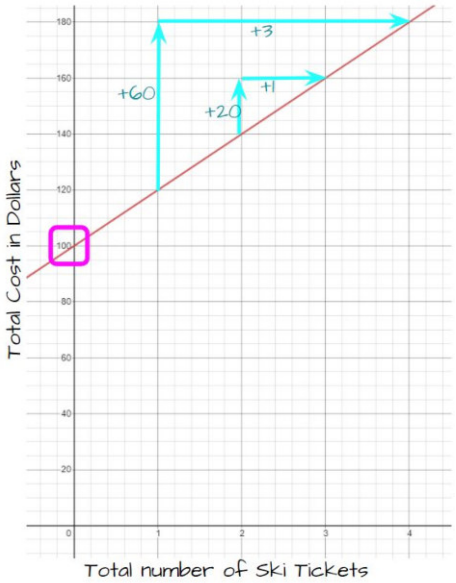
Rate of Change
An increase of 80 for every 4 x's
= an increase of 20 for every 1 x

⬇

$$\frac{\Delta y}{\Delta x} = \frac{80}{4} = \frac{20}{1}$$

Graphs

Initial Value of **100** is found when x=0, **on the y-axis**



Rate of Change
For every 3 x's, y increases by 60 so...
For every 1 x, y increases by 20

Equations

Even when x is 0, you still will have to pay $100 so your **Initial Value** is **100**

$$y = 20x + 100$$

For every 1x, y increases by 20 ...
So your **Rate of Change** is 20