

Number

Additive Reasoning



PreK (4-5 years old)	Kindergarten	Grade One	Grade Two
<p>Understanding of number values and sequences to 10 <i>(counting, cardinality, conservation, and stable order)</i></p> <p>1:1 Correspondence</p>	<p>Understanding of number values and sequences to 20 <i>(counting, cardinality, and stable order)</i></p> <p>1:1 Correspondence</p> <p>Comparing quantities</p>	<p>Understanding of number values and sequences to 120 <i>(cross century, cross decade)</i></p> <p>Understanding place value when adding and subtracting numbers within 100 <i>(in context and in equations)</i></p>	<p>Use place value understanding to add and subtract numbers accurately, flexibly, efficiently, and strategically within 1,000 <i>(in context and in equations)</i></p> <p>(NO standard algorithm)</p>
Models for Intervention			
5- and 10-frames for counting	10 frames	Multiple 10 frames	Multiple 10 frames/strips Number lines/adding up <i>(using part-part-total focus)</i> . These start from zero.
Models for Instruction			
Fingers 5-frames for subitizing Dot patterns <i>(regular and irregular)</i>	Fingers Dot patterns <i>(regular and irregular)</i> Bead Racks	<u>Number</u> - number paths <u>Adding and Subtracting</u> - multiple 10 frames, ten strips	Place value materials <i>(e.g., base ten blocks/pieces, digi-blocks, 10 frames, 10 strips)</i> Number lines <i>(articulated)</i>
Critical Strategies			
<p>Counting by 1s Subitizing Organizing¹ Tracking²</p>		<p>Compose, Decompose, & Recompose using Place Value:</p> <ul style="list-style-type: none"> Decompose both numbers to add/subtract (Recompose like units) Decompose one number and add/subtract by place value units. <p>Approach subtraction as a missing addend problem. (Understand addition and subtraction are related operations.)</p> <p>Use compensation based on the commutative, identity, and associative properties</p>	

¹Young children need to have a way to keep track of what's been counted and what still needs to be counted. How do they organize to keep track without a teacher saying "line them up and touch them" to count?

²Tracking refers to the actual gesture of touching and counting.

Multiplicative Reasoning

Fractions



Grade Three	Grade Four	Grade Five
<p>Multiply and divide numbers within 100 <i>(in context and in equations)</i></p>	<p>Multiply and divide any two numbers within 1,000 <i>(in context and in equations)</i></p>	<p>All four operations with fractions <i>(in context and in equations)</i></p> <p>(NO standard algorithms – using modeling and/or decomposition approaches.)</p>
Models for Intervention		
<p>Strong connections between grouping and area models.</p> <p>Use of area models for multiplication facts.</p>	<p>Area models to support decomposition for multiplication.</p> <p>Partitive (sharing) models for division.</p>	<p>Area models to build equivalence for add/sub.</p> <p>Parts/whole models for multiplication, with a focus on whole numbers x fractions.</p> <p>Measurement models for division of fractions.</p>
Models for Instruction		
<p>Grouping models (i.e. circles and stars, loops and groups, beans and cups), jumps on a number line, repeated addition, skip counting; area models for products to 100 (may start by using place value blocks)</p> <p>Experience with both partitive (sharing) models and quotitive (partial quotients) models</p>	<p>Area models for products OR quotients to 1,000</p> <p>The use of area models to develop decomposition strategies for multi-digit computation</p> <p>Experience with both partitive (sharing) models and quotitive (partial quotients) models both in equations and in context.</p>	<p>Area models for part/whole relationships, place value blocks, Cuisenaire rods, fraction bars, fraction pieces, geoboards, pattern blocks</p>
Critical Strategies		
<p>Compose and Decompose using factors (initially 2s, 5s, 10s)</p> <p>Use compensation based on the commutative, identity, associative, and distributive properties</p> <p>Approach division as a missing factor in a multiplication problem. (Understanding how multiplication and division are related operations.)</p>		<p>Model and Identify equivalent fractions</p> <p>Connect and apply previous understanding of whole number operations and properties of addition and multiplication while operating with fractions.</p>

Proportional Reasoning		Expressions & Equations		Linear Relationships
Grade Six	Grade Seven	Grade Seven	Grade Eight	Grade Eight
Use models to compare ratios, and solve problems including those involving unit rates	Solves proportional reasoning problems using a model and relationship of multiplication and division <i>(The cross products algorithm does not qualify as demonstrating understanding.)</i>	Operate with signed numbers and create equivalent expressions	Solve equations for unknowns which may include signed numbers.	Understand linear relationships using tables, and/or graphs, and/or equations. Make connections among representations of linear relationships.
Models for Instruction				
Concrete models (i.e. tiles) Diagrams Percent bar	Tape diagrams Double number line Geometric figures	2 sided chips Floats and Anchors Algebra Tiles Number Lines	Algebra Tiles Hangar Diagrams Pan Balances Inverse Operations	Graphing on a coordinate plane - including technology (Desmos, GeoGebra)
Critical Strategies				
Look for and identify multiplicative relationships in tables and diagrams. Focus on the meaning of the quantities in a situation.		Zero pairs are useful tools when working with signed numbers	Use inverse operations for solving problems	Find the linear rate of change between two quantities/x and y.
Meaning				
Students show an understanding of ratios and rates including unit rates involving whole numbers or fractions. They identify equivalent ratios and use unit rates to solve problems. They can demonstrate this understanding using models and expressions.	Students will extend the basic understanding of ratios using proportions ($a/c=b/d$). This means applying proportions to topics including percentages, similarity, scaling, conversion, etc. This work builds on, and extends, the work in proportional reasoning from Grade 6.	Students will begin to build their understanding of simplifying expressions with models. Students will show understanding of all four basic operations with integers	Students will continue to use models to extend their understanding of equivalent expressions into solving one variable equations. Using models will support conceptual understanding of solving algebraic equations and allow students to move into solving equations where there are infinite solutions, no solutions and non-integer solutions.	Students show their understanding of linear relationships by representing them in four ways: verbal, tables, graphs, and equations.