Multiplicative Reasoning

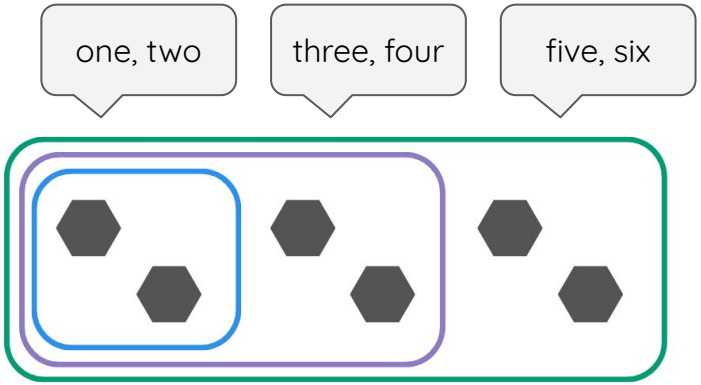
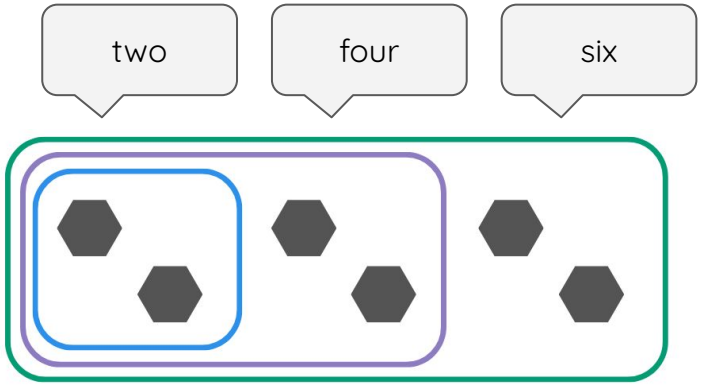
Grade Three HLC

Multiply and divide within 100 within context and with equations.

September → **Grade Three Learning Progressions** → June

Students must use models to build understanding along this trajectory and interact with a variety of contexts for multiplication and division. Models should support students' ability to unitize–understand a group or collection of items represents "one." (For example, one group of 5 consists of 5 individual items but is classified as one group.)

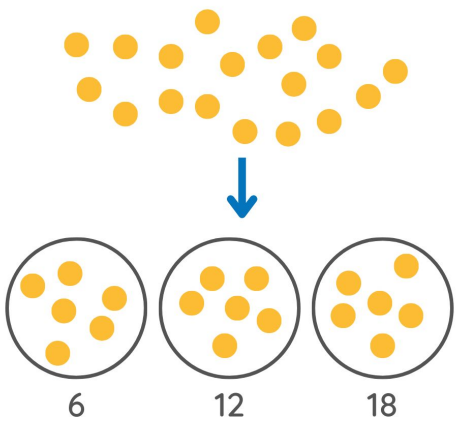
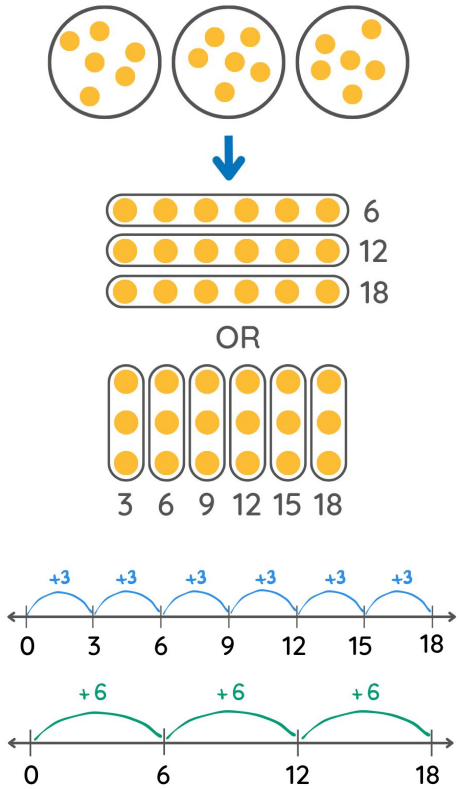
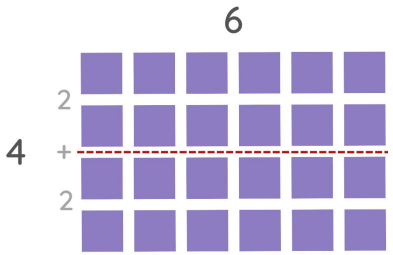
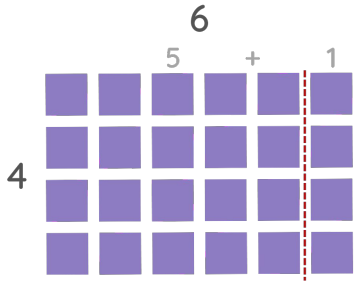
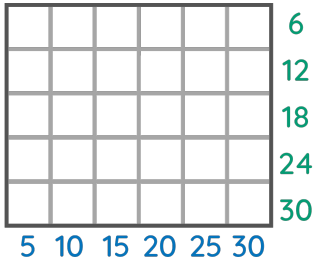
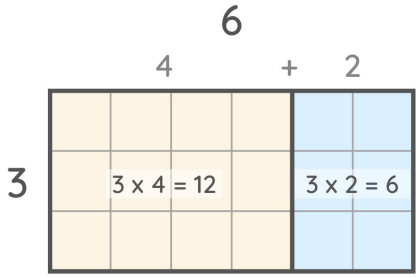
Counting by Equal Groups (Unitizing) to Build Multiplicative Understanding *(modeling and then counting by 1s or skip counting)*

Counts by ones in equal sized subgroups; counts individual objects within the group.	Skip counts the equal sized groups or uses repeated addition to tell the cumulative total of each group (no longer counts individual objects, but counts equal groups).
	

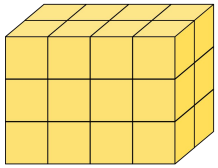
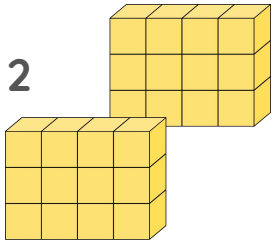
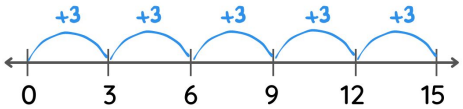
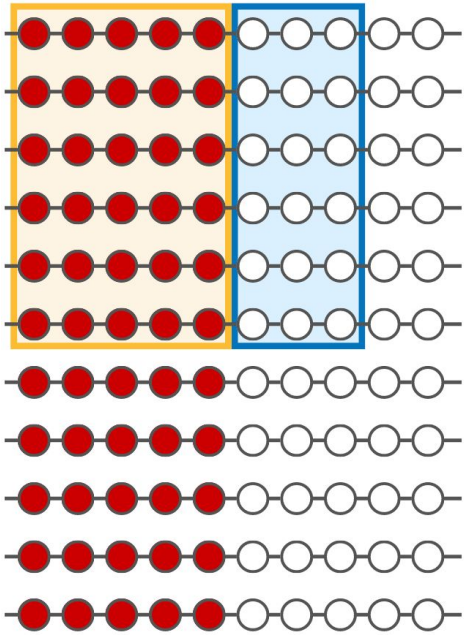
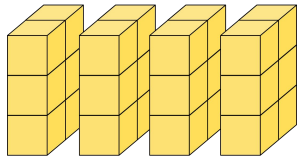
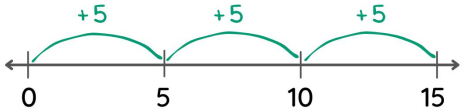
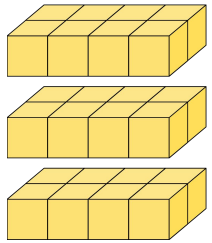
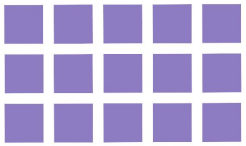
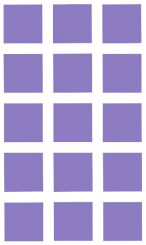
Operations: Multiplication and Division

Students must use models to build understanding along this trajectory and interact with a variety of contexts for multiplication and division. Models should support students developing understanding of the magnitude of digits in their place values. In Grades 1 and 2, students thought about place value as follows: 245 = 200 + 40 + 5. In Grades 3 and 4, place value understanding becomes multiplicative: 245 = 2(100) + 4(10) + 5(1)
Students also use relational thinking when composing, decomposing and recomposing

Multiplication - Composition and Decomposition

Organizes a collection into equal groups.	Organizes equal groups into rows and columns and skip counts by rows OR columns. Organizes equal groups as jumps on a number line.	Uses an array to decompose into smaller arrays.	Uses the area model for products to 100 (10x10) to understand length and width as dimensions that are 1x1 square units.
 6 + 6 + 6 = 18		 4 x 6 = (2 x 6) + (2 x 6) 2 x 6 = 12 12 + 12 = 24 4 x 6 = 24  4 x 6 = (4 x 5) + (4 x 1) 4 x 5 = 20 20 + = 24 4 x 6 = 24	**Skip counts rows or columns**  **Decomposes side lengths using the distributive property**  3 x 6 = (3 x 4) + (3 x 2) 12 + 6 = 18 3 x 6 = 18

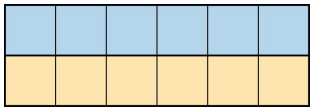
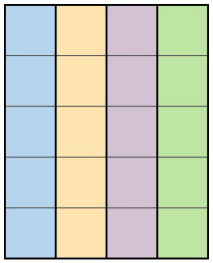
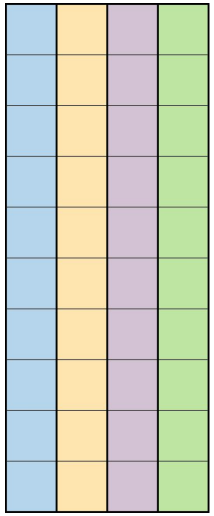
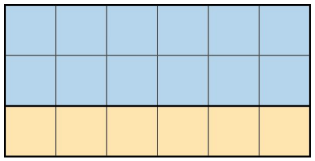
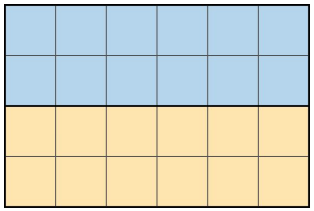
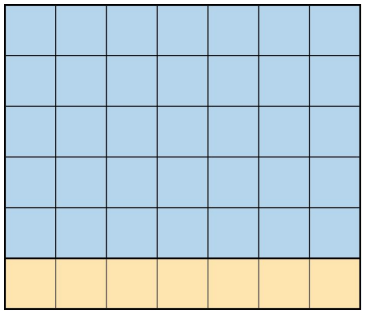
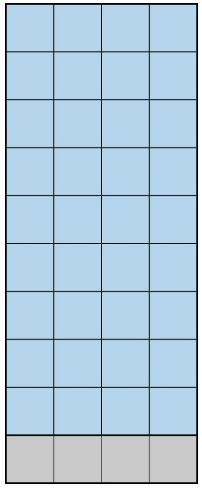
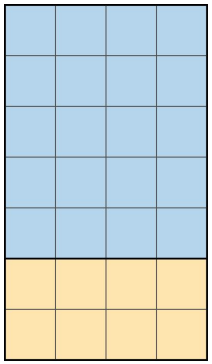
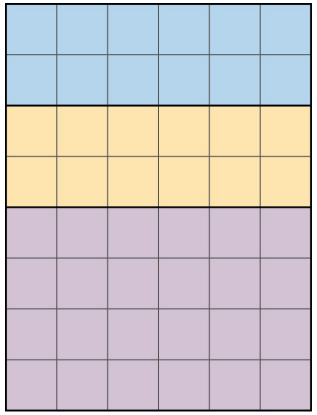
Properties of Multiplication *These properties are investigated throughout the year with different numbers and problem situations. The sequence of how the properties appear below does not suggest the order in which to explore them. Many times the properties can be explored simultaneously with student work.)*

Identity Property	Commutative Property	Distributive Property	Associative Property
7 x 1 = 7	3 x 5 = 5 x 3	6 x 8 = (6 x 5) + (6 x 3)	4 x 3 x 2
	 3 + 3 + 3 + 3 + 3 = 5 + 5 + 5	Eight groups of six is the same value as six groups of five plus six groups of three.	 I can multiply the dimensions in any order to find the volume.
I have seven groups of one, which is the same as seven.	Three groups of five is the same value as five groups of three.	30 + 18 = 48	(4 x 3) x 2 12 x 2 24 
			 4 x (3 x 2) 4 x 6 24
		*100 bead rack illuminates benchmark numbers*	(4 x 2) x 3 8 x 3 24 
	3 x 5  3, 6, 9, 12, 15	5 x 3  5, 10, 15	

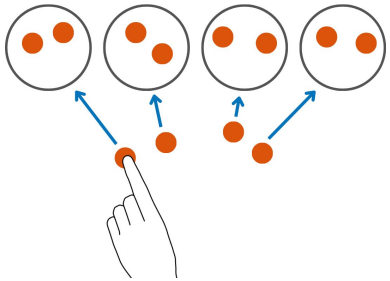
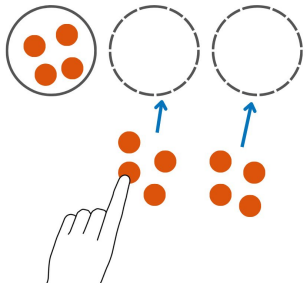
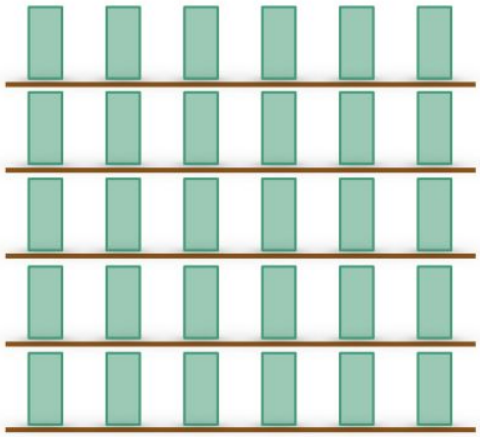
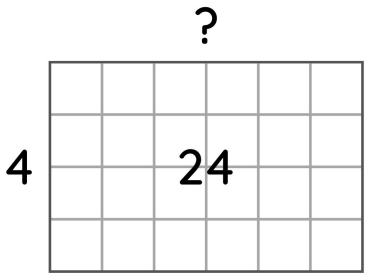
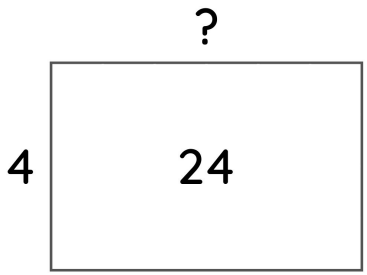
Developing Multiplication Fact Strategies

Fact fluency must develop **through use of models,** NOT through rote memorization. Students simultaneously explore properties of multiplication through composition and decomposition which build relational thinking strategies.

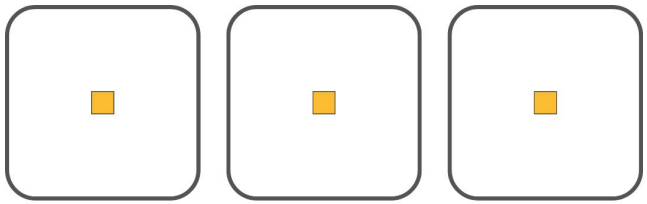
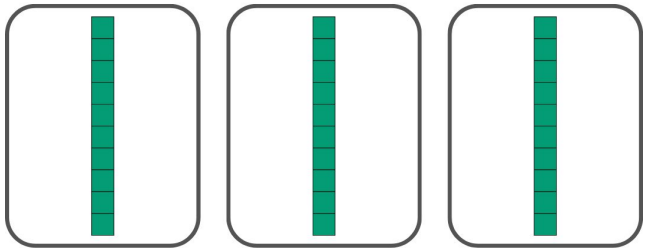
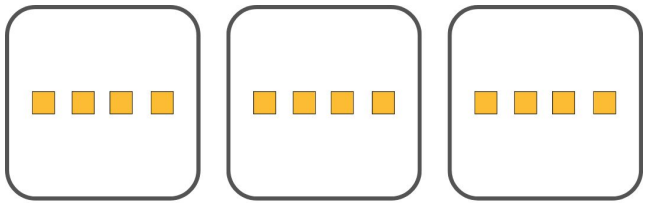
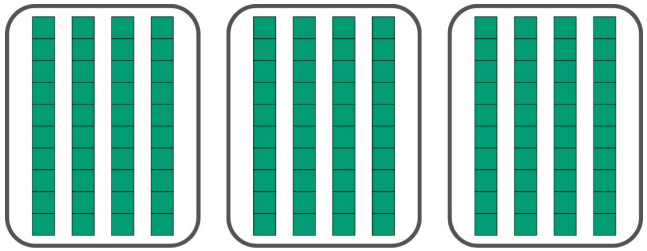
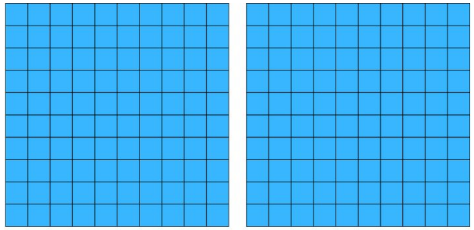
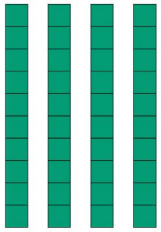
Below we show **examples of how students **might** derive multiplication facts. These examples are not meant to prescribe certain strategies that must be used.*

Practices and uses known facts including **0**, **1s**, **2s**, **5s** and **10s** facts (referred to as foundational facts)	Derives **3s** facts and **4s** facts using foundational facts, or other derived facts.	Derives **6s**, **9s** facts using foundational facts or other derived facts.	Derives **7s** and **8s** facts using foundational facts or other derived facts.
2 x 6 = 6 + 6  **Multiplying by 2** is like adding the same number twice (doubling). I can quickly **multiply by 5 or 10** by skip counting.  5, 10, 15, 20 **Multiplying by 10** is based on place value.  4 tens = 40 10, 20, 30, 40	3 x 6 = (2 x 6) + 6  **To multiply by 3,** I can just double the other factor and then add one more group. **To multiply by 4,** I can just double the other factor and then double it again.  4 x 6 = 2 x (2 x 6)	6 x 7 = (5 x 7) + 7  **To multiply by 6,** I can just multiply the other factor by 5 and then add one more group. **To multiply by 9,** I can just multiply the other factor by 10 and then subtract one group.  9 x 4 = (10 x 4) - 4	7 x 4 = (5 x 4) + (2 x 4) **To multiply by 7,** I can decompose into parts I know, like 5 and 2, and then add them together.   **To multiply by 8,** I can just double the other factor three times. 8 x 6 = 2 x 2 x (2 x 6)

Division - Composition and Decomposition *Students model both partitive and quotitive situations.*

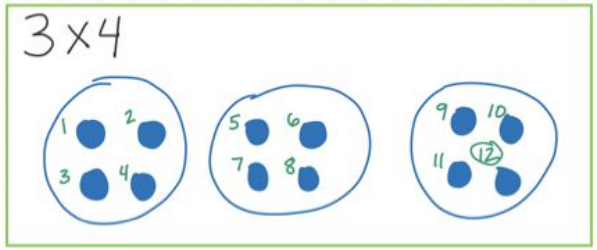
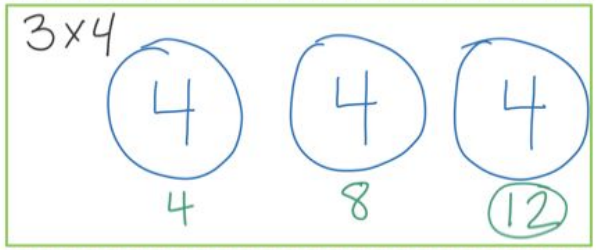
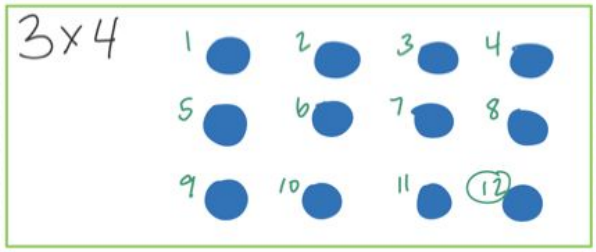
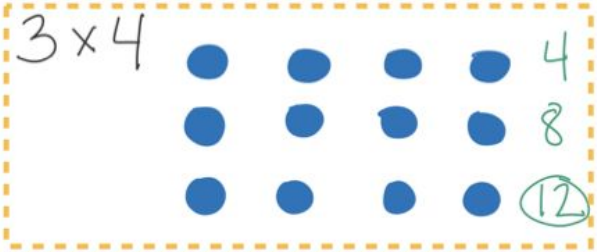
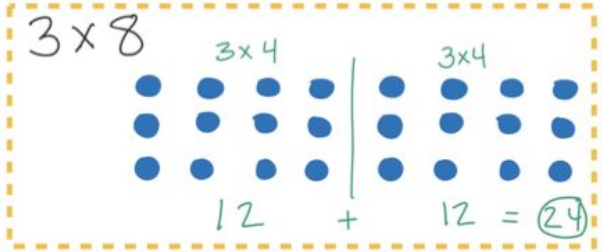
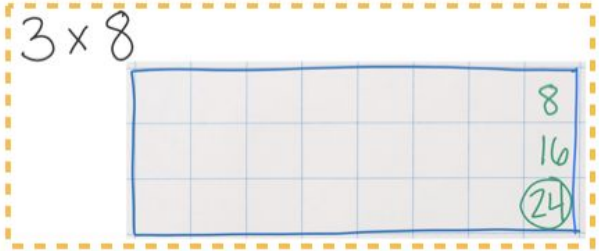
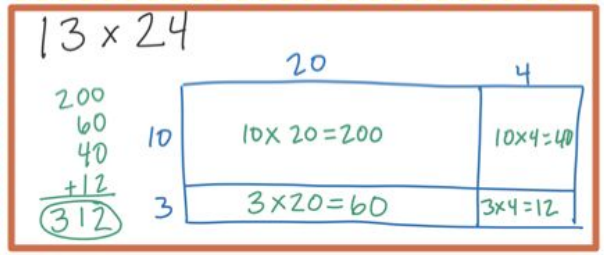
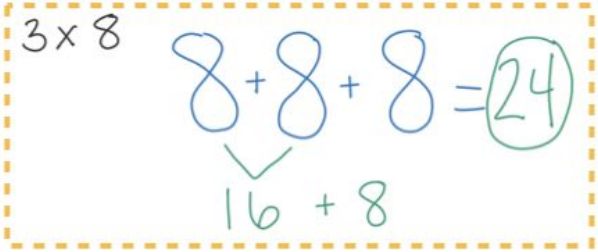
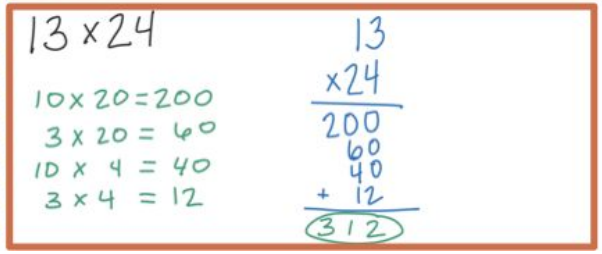
Shares by ones into equal sized subgroups- shares individual objects until all items are shared in groups equally (partitive division). Uses repeated subtraction of equal size groups or sharing in larger chunks (sharing by 2s, 3s, 4s and 5s) until all of the items have been removed from the total (quotitive division).	Organizes groups into rows or columns based on total number of objects and the given number of rows or columns. Students will start to use the missing factor as the answer.	Uses the area model to determine missing side length (missing dimension) through 10 x 10. Uses inverse relationship, and considers the missing factor problem for multiplication to solve a division problem.
Partitive I have 12 cookies to share equally across 4 plates. **How many cookies on each plate?** 12 ÷ 4 = # in each group (12: in all; 4: groups)  ***Quotative*** I have 12 cookies. Each plate holds 4 cookies. **How many plates do I have?** 12 ÷ 4 = # of groups (12: in all; 4: in each group) 	I have 30 books to organize on 5 shelves. **How many books are on each shelf?** 30 ÷ 5 = ___ 5 x ___ = 30 How many books per shelf? 5 rows (shelves) 	I want to build a garden that is 24 sq. feet. One side of the garden will be 4 feet long. **How long does the other side need to be?**  24 ÷ 4 = ___  4 x ___ = 24

Composing and Decomposing Using Base Ten Units and Place Value - 1s, 10s, 100s *(Students must use models to build understanding along this trajectory. Models should support students developing understanding of the magnitude of digits in their place values.)*

Uses place value understanding to multiply single digit times 10. This involves extending understanding of ***single digit* x *single digit*** to ***single digit* x *a group of ten***	Uses place value understanding to multiply a single digit by multiple of 10. This involves extending understanding of ***single digit* x *single digit*** to ***single digit* x *multiple of ten***	Students decompose any number through expanded notation.
3 groups of 1 3 x 1 = 3  *so I know…* 3 groups of 10 3 x 1 ten = 3 tens = 30 3 x 10 = 30 	3 groups of 4 3 x 4 = 12  *so I know…* 3 groups of 40 3 x 4 tens = 12 tens = 120 3 x 40 = 120 	247 = (2 x 100) + (4 x 10) + (7 x 1)  2 x 100  4 x 10  7 x 1

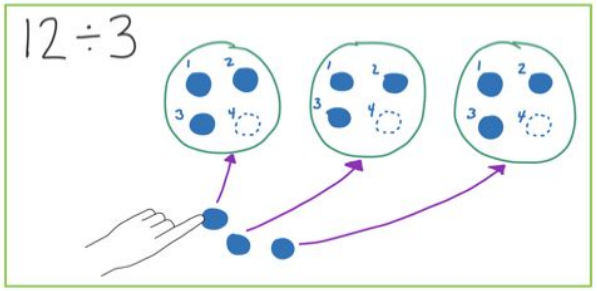
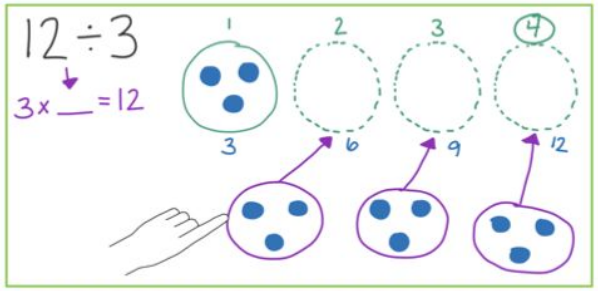
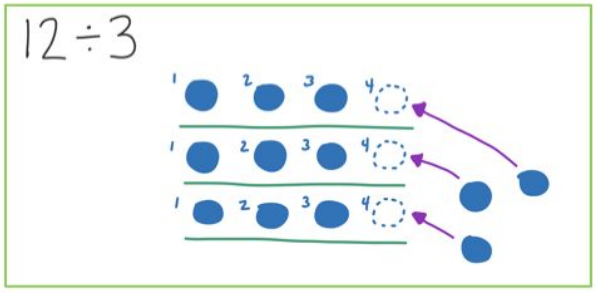
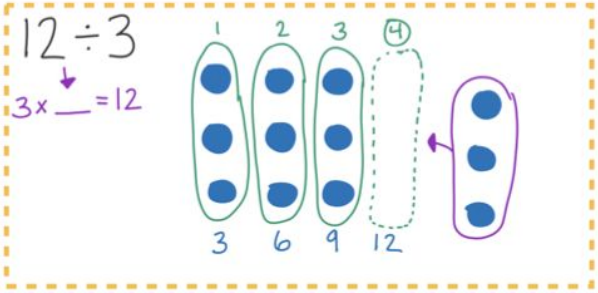
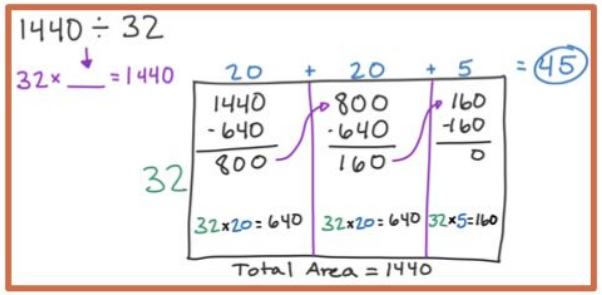
Models and Strategies for Multiplication (Across Grades 3-4)

Grade 3 | Grade 3 + 4 | Grade 4

Models	Strategies: Group + count by ones	Strategies: Skip Count	Strategies: Partial Products (Decomposition + Recomposition)
Equal Groups			Equal Groups are not an appropriate model for this strategy.
Array			
Area Model			
Equations	Equations are not an appropriate model for this strategy.		

Models and Strategies for Division (Across Grades 3-4)

Grade 3 | Grade 3 + 4 | Grade 4

Models	Strategies: Grouping/Fair Share	Strategies: Repeated Subtraction or Skip Counting	Strategies: Partial Quotients (Decomposition + Recomposition)
Equal Groups			Equal Groups are not an appropriate model for this strategy.
Array or Area Model			
Equations	Equations are not an appropriate model for this strategy.