

Additive Reasoning

Grade Two HLC

Use place value understanding to add and subtract numbers accurately, flexibly, efficiently, and strategically within 1,000
(in context and in equations) **(NO standard algorithm)**

September



Grade Two Learning Progressions



June

Students must use models to build understanding of the HLC and interact with a variety of contexts.

Rote Oral Count Sequence (rote counting from 1; rote counting from any start number)

Teachers need to purposefully choose a variety of number ranges including opportunities to practice teen numbers, crossing decades, and centuries. This information is often best collected in student interviews checking on clusters of 5 numbers at various starting points.

Counts Forward (FWD) and Backward (BWD) within the range **1-120** starting at any number

Counts FWD and BWD within the range **1-220** starting at any number

Counts FWD and BWD within the range **1-500** starting at any number

Counts FWD and BWD within the range **1-1000** starting at any number

Skip counts FWD and BWD by 10s **starting at any number** within the range **1-120**

Skip counts FWD and BWD by 10s **on decade** within the range **1-1000**

Skip counts FWD and BWD by 10s starting at any number within the range **1-500**

Skip counts FWD and BWD by 10s starting at any number within the range **1-1000**

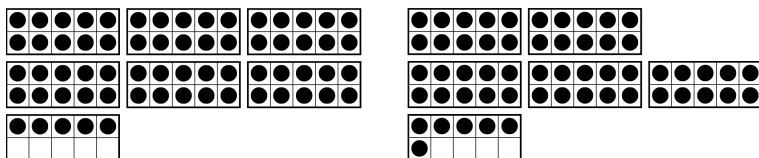
Skip counts FWD and BWD by 100s starting on century within the range **1-1000**

Skip counts FWD and BWD by 100s starting at any number within the range **1-1000**

Ordering & Magnitude

Uses place value understanding to compare 2-digit numbers.

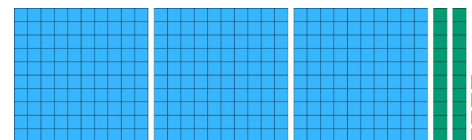
65 is greater than 56. 65 has 1 more ten.



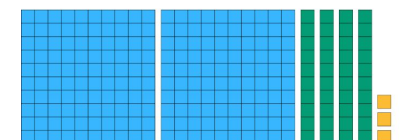
56 is less than 65. 56 has 1 fewer ten.

Uses place value understanding to compare 3-digit numbers.

324 is greater than 243. 324 has 1 more hundred.

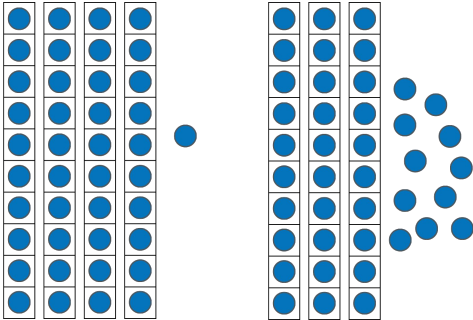
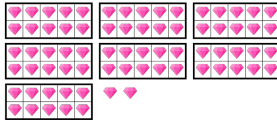
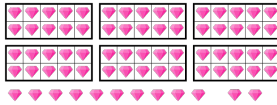
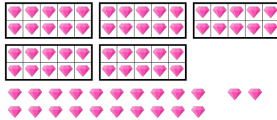
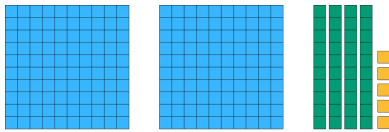
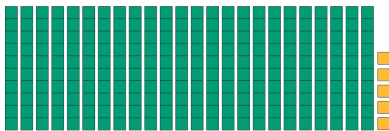
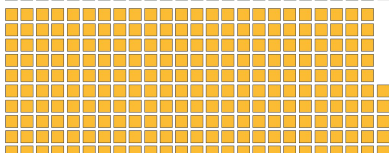
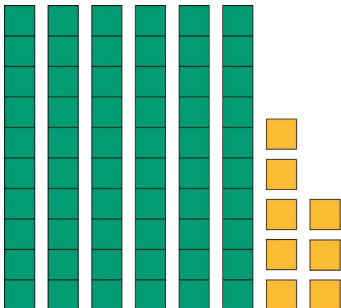
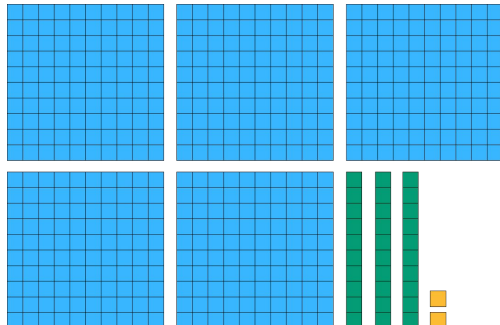


243 is less than 324. 243 has 1 fewer hundred.

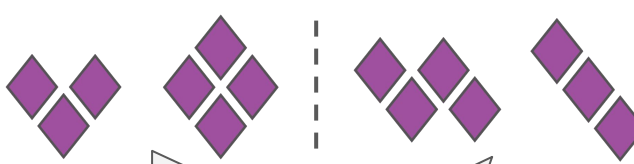
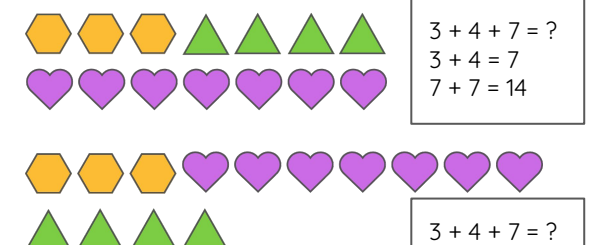
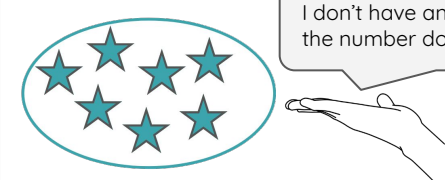


Operations: Addition and Subtraction Students must use models to build understanding along this trajectory and interact with a variety of contexts for addition and subtraction. Models should support students developing understanding of the magnitude of digits in their place values.

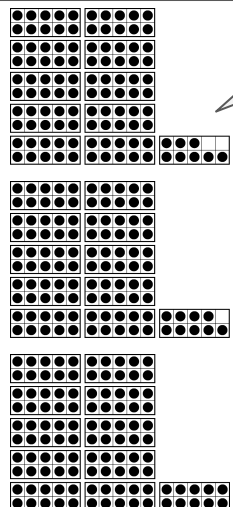
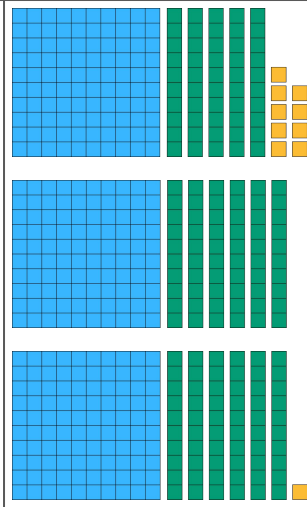
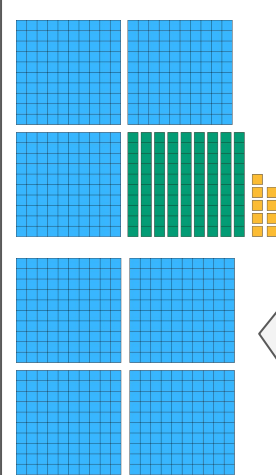
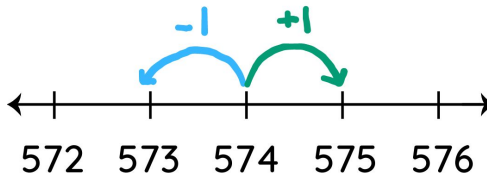
Composition, Decomposition Students must use models to build understanding and flexibility when composing and decomposing quantities. Students must use models to build understanding of unitizing: 10 ones = 1 ten; 10 tens = 1 hundred, etc. as well as *equivalent* representations of a specific quantity (i.e 126 is simultaneously 126 ones; 12 tens and 6 ones; 1 hundred, 2 tens, and 6 ones; 1 hundred and 26 ones; 11 tens and 16 ones; 9 tens and 36 ones; etc.)

All numbers within the range 1-50	All numbers within the range 1-100	All numbers within the range 1-1000														
<div></div> <div>41 is... 4 tens and 1 one AND 3 tens and 11 ones.</div>	<div>72 can look like...</div> <div><div>7 tens 2 ones</div></div> <div><div>6 tens 12 ones</div></div> <div><div>5 tens 22 ones</div></div>	<div>I can show 245 as...</div> <div><div>2 hundred, 4 tens, and 5 ones.</div></div> <div><div>24 tens and 5 ones.</div></div> <div><div>245 ones</div></div>														
Builds and tells the value of the digits in any 2-digit number. Decompose any 2-digit number into its place value parts.		Builds and tells the value of the digits in any 3-digit number. Decompose any 3-digit number into its place value parts.														
<div></div> <div><table><tr><td>6</td><td>0</td></tr></table><table><tr><td>8</td></tr></table></div> <div><table><tr><td>6</td><td>8</td></tr></table></div>		6	0	8	6	8	<div></div> <div><table><tr><td>5</td><td>0</td><td>0</td></tr></table><table><tr><td>3</td><td>0</td></tr></table><table><tr><td>2</td></tr></table></div> <div><table><tr><td>5</td><td>3</td><td>2</td></tr></table></div>	5	0	0	3	0	2	5	3	2
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6	8															
5	0	0														
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2																
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Properties of Addition These properties are investigated throughout the year with different numbers and problem situations. The sequence of how the properties appear below does not suggest the order in which to explore them. Many times the properties can be explored simultaneously with student work.

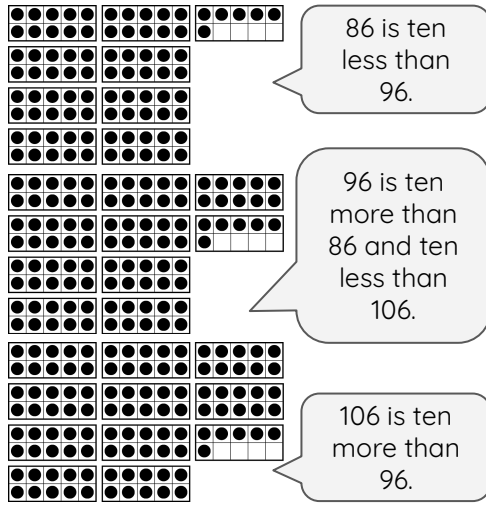
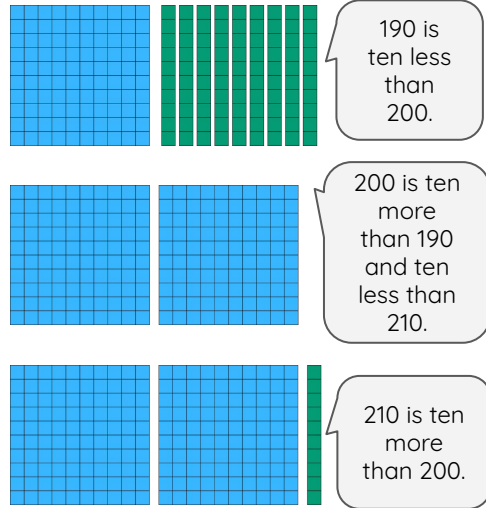
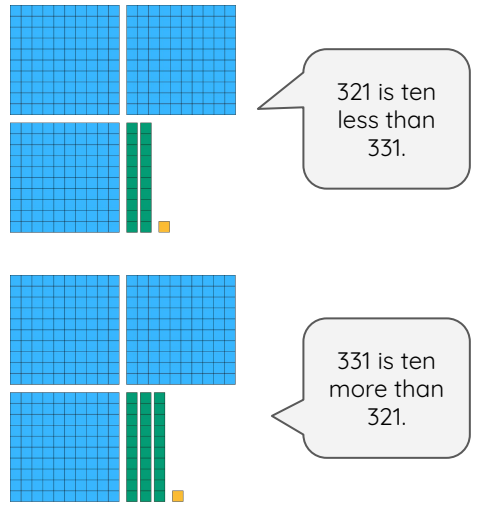
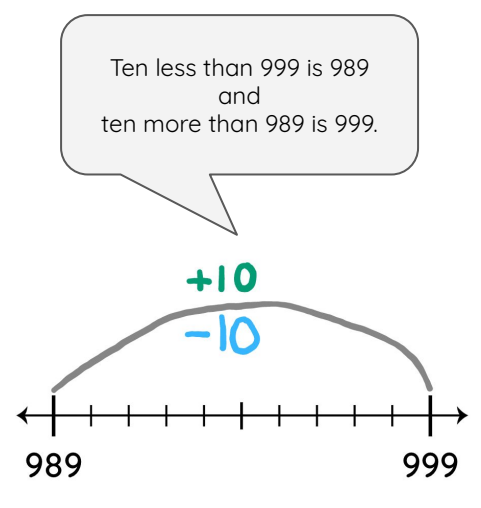
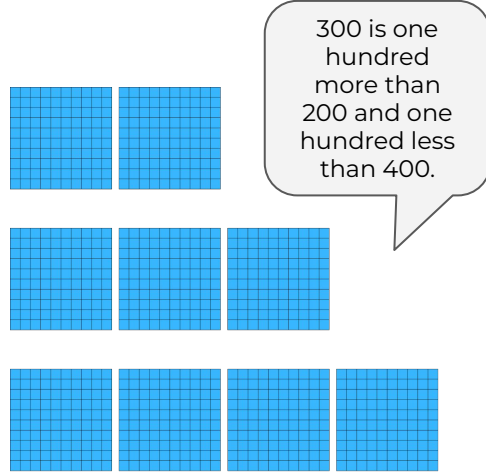
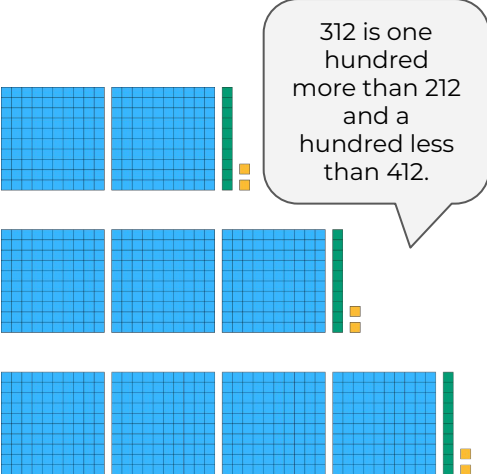
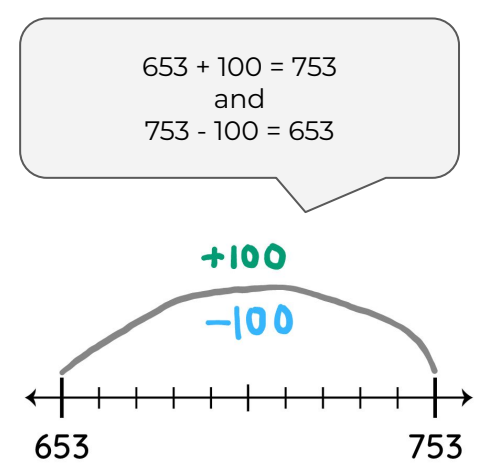
Commutative Property	Associative Property	Identity Property
 <p>3 and 4 makes 7</p> <p>4 and 3 makes 7, too</p> <p>It doesn't matter what order you put them together.</p> <p>$3 + 4 = 4 + 3$</p>	<p>$3 + 4 + 7$</p>  <p>$3 + 4 + 7 = ?$ $3 + 4 = 7$ $7 + 7 = 14$</p> <p>$3 + 4 + 7 = ?$ $3 + 7 = 10$ $10 + 4 = 14$</p>	<p>When you add zero to any number, the number stays the same.</p>  <p>I don't have anything to add so the number doesn't change.</p>

Place Value - Building Understanding Students must use models to build understanding along this trajectory and interact with a variety of contexts for addition and subtraction. Models should support students developing understanding of the magnitude of digits in their place values.

Models the number 1 more/1 less within 120	Models the number 1 more/1 less within 220	Models the number 1 more/1 less within 500	Models the number 1 more/1 less within 1000
 <p>108 is one less than 109.</p> <p>109 is one less than 110 and one more than 108.</p> <p>110 is one more than 109.</p>	 <p>159 is one less than 160.</p> <p>160 is one less than 161 and one more than 159.</p> <p>161 is one more than 160.</p>	 <p>399 is one less than 400.</p> <p>One more than 399 completes the hundred to make 400.</p>	<p>One less than 574 is 573. One more than 574 is 575.</p> 

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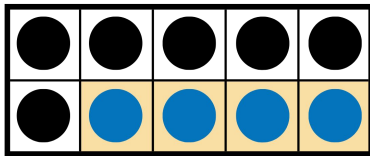
Place Value - Building Understanding *(cont.)*

Models the number 10 more/10 less from any number within 120	Models the number 10 more/10 less from any number within 220	Models the number 10 more/10 less from any number within 500	Models the number 10 more/10 less from any number within 1000
 <p>86 is ten less than 96.</p> <p>96 is ten more than 86 and ten less than 106.</p> <p>106 is ten more than 96.</p>	 <p>190 is ten less than 200.</p> <p>200 is ten more than 190 and ten less than 210.</p> <p>210 is ten more than 200.</p>	 <p>321 is ten less than 331.</p> <p>331 is ten more than 321.</p>	 <p>Ten less than 999 is 989 and ten more than 989 is 999.</p> <p>+10 -10</p> <p>989 999</p>
	Models the number 100 more/100 less from any century within 1000	Models the number 100 more/100 less from any number within 500	Models the number 100 more and 100 less from any number within 1000
	 <p>300 is one hundred more than 200 and one hundred less than 400.</p>	 <p>312 is one hundred more than 212 and a hundred less than 412.</p>	 <p>653 + 100 = 753 and 753 - 100 = 653</p> <p>+100 -100</p> <p>653 753</p>

Developing and Extending Fact Fluency

Students use relational thinking to develop fact fluency within 10 and then extend those fact patterns to greater numbers.

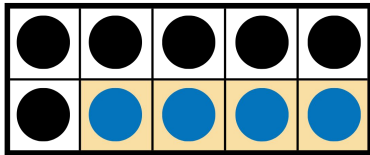
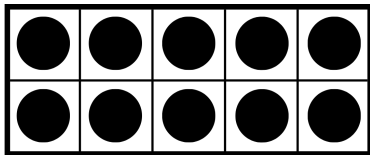
Uses understanding of combinations to 10 to find combinations to 20.



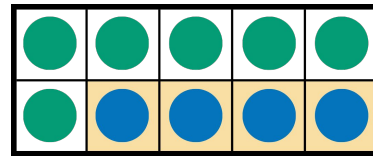
$$6 + 4 = 10$$

SO

$$16 + 4 = 20$$



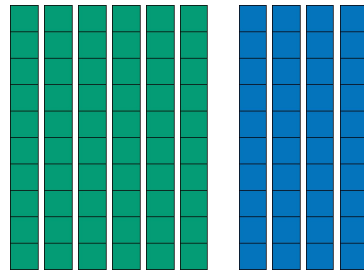
Uses understanding of combinations to 10 to find multiple of 10s partners to 100.



$$6 + 4 = 10$$

SO

$$60 + 40 = 100$$



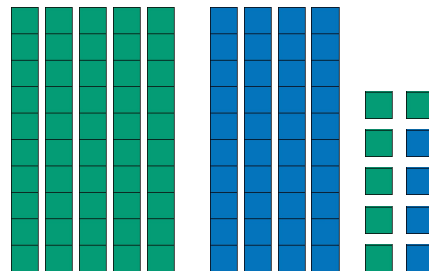
$$56 + \underline{\quad} = 100$$

$$50 + 40 = 90$$

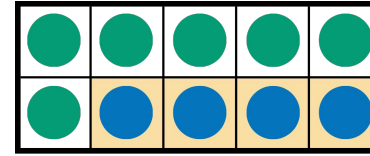
$$6 + 4 = 10$$

$$90 + 10 = 100$$

$$56 + 44 = 100$$



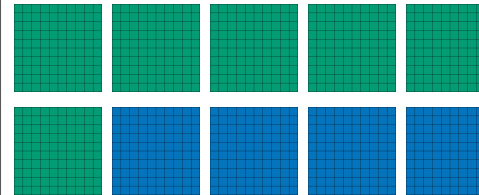
Uses understanding of combinations to 10 to find multiple of 100s partners to 1000.



$$6 + 4 = 10$$

SO

$$600 + 400 = 1000$$



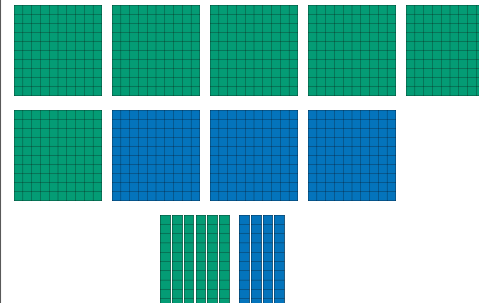
$$560 + \underline{\quad} = 1000$$

$$500 + 400 = 900$$

$$60 + 40 = 100$$

$$900 + 100 = 1000$$

$$560 + 440 = 1000$$



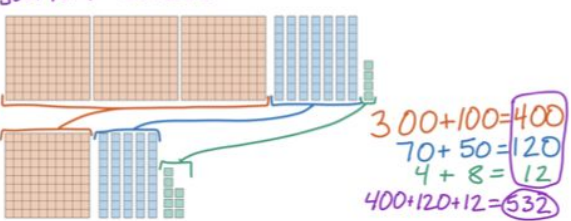
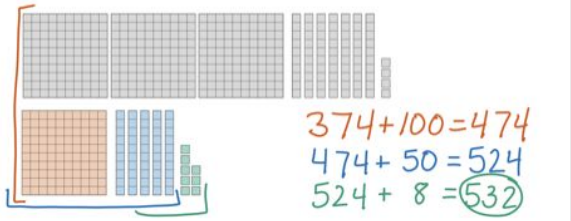
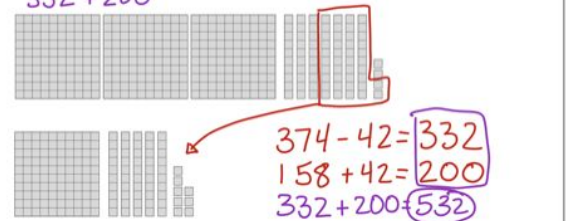
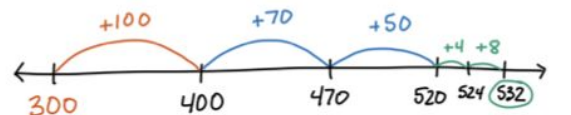
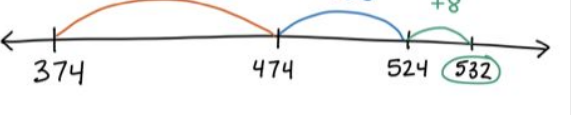

Uses understanding of combinations to 1s, 10s, 100s to add any numbers within 1000.

See model/strategy charts on the following pages for examples of adding and subtracting within 1000.

Use Place Value to compose, decompose and recompose

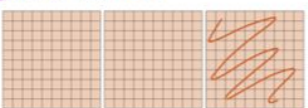

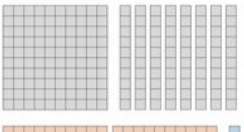

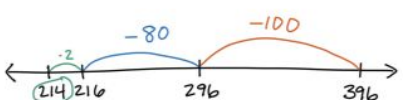

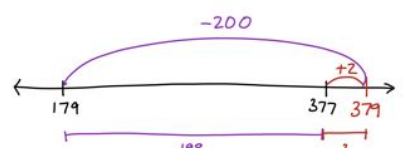
Decompose both numbers to add and subtract, decompose one number to add and subtract, recompose like units, missing addend, compensation
There is an explicit connection between counting and addition (i.e. counting 10 more is the same as adding 10, counting back 10 is the same as subtracting 10).

Models & Strategies for Addition

Strategies	
Place Value: Decompose both numbers	
Place Value: Decompose one number	
Compensation	
Models	Ten Frames
	Ten Frames are not an efficient model for 3-digit computation. See Grade 1 Progression for examples with 2-digit numbers.
Models	Place Value Materials
	$374 + 158$ $300 + 70 + 4$ $100 + 50 + 8$  $300 + 100 = 400$ $70 + 50 = 120$ $4 + 8 = 12$ $400 + 120 + 12 = 532$
	$374 + 158$ $100 + 50 + 8$  $374 + 100 = 474$ $474 + 50 = 524$ $524 + 8 = 532$
	$374 + 158$ -42 $+42$ $332 + 200$  $374 - 42 = 332$ $158 + 42 = 200$ $332 + 200 = 532$
Models	Number Lines
	$374 + 158$ $300 + 70 + 4$ $100 + 50 + 8$ $300 + 100 = 400$ $400 + 70 = 470$ $470 + 50 = 520$ $520 + 4 = 524$ $524 + 8 = 532$ 
	$374 + 158$ $100 + 50 + 8$ $374 + 100 = 474$ $474 + 50 = 524$ $524 + 8 = 532$ 
	$374 + 158$ -42 $+42$ $332 + 200$ $374 - 42 = 332$ $158 + 42 = 200$ $332 + 200 = 532$ 

*We recommend starting with articulated number lines in Grade 2, and then connecting them to open number lines while moving from 2-digit to 3-digit computation.

Models & Strategies for Subtraction

Strategies	
Place Value: Decompose both numbers	
Place Value: Decompose one number	
Missing Addend	
Compensation	
Models	Ten Frames
	Ten Frames are not an efficient model for 3-digit computation. See Grade 1 Progression for examples with 2-digit numbers.
Place Value Materials	$396 - 182$ $300 + 90 + 6$ $100 + 80 + 2$  $300 - 100 = 200$ $90 - 80 = 10$ $6 - 2 = 4$ $200 + 10 + 4 = 214$
	$396 - 182$ $100 + 80 + 2$  $396 - 100 = 296$ $296 - 80 = 216$ $216 - 2 = 214$
Number Lines	$396 - 182 \rightarrow 182 + \underline{\quad} = 396$  $182 + 200 = 382$ $382 + 10 = 392$ $392 + 4 = 396$ $200 + 10 + 4 = 214$
	$377 - 198$ $379 - 200$  $377 + 2 = 379$ $198 + 2 = 200$ $379 - 200 = 179$
Number Lines	<p>Number Lines are not an appropriate model for this strategy.</p>
	$396 - 182$ $100 + 80 + 2$  $396 - 100 = 296$ $296 - 80 = 216$ $216 - 2 = 214$
	$396 - 182$ $182 + \underline{\quad} = 396$  $182 + 200 = 382$ $382 + 10 = 392$ $392 + 4 = 396$ $200 + 10 + 4 = 214$
	$377 - 198$ $379 - 200$  $377 + 2 = 379$ $198 + 2 = 200$ $379 - 200 = 179$

*We recommend starting with articulated number lines in Grade 2, and then connecting them to open number lines while moving from 2-digit to 3-digit computation.