CHAPTER 4

Identifying Where Students Are

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Formative Assessment in Math Class

The intention of this chapter is to identify some approaches to discovering students current thinking about math concepts and suggest possible next instructional steps to move them along their path to deeper understanding. The process of identifying where students are---and providing next instructional steps---is the heart of formative assessment and essential to ensuring success for all learners.

Introduction

Central to the practices described in earlier chapters is the notion that teachers provide instructional experiences that move students along a pathway of learning. While we can identify many landmarks that students will need to pass along the way, each student's exact pathway will be unique. The intention of this chapter is to identify some approaches to discovering students current thinking about math concepts and suggest possible next instructional steps to move them along their path to deeper understanding. The process of identifying where students are and providing next instructional steps is the heart of formative assessment and essential to ensuring success for all learners. Benjamin Bloom (1969) has said, "Evaluation which is directly related to the teaching-learning process as it unfolds can have highly beneficial effects on the learning of students, the instructional process of teachers, and the use of the instructional materials by teachers and learners" (Bloom, 1969, p. 50). Our focus for this chapter is to investigate and provide evidence for the routine use of formative assessment to tailor instruction to meet the needs of all students.

What is Formative Assessment?

There are many forms of assessment that can and do take place during classroom instruction. We see two main forms of assessment that provide data or evidence of students' understanding of mathematics. If the teacher's intention is to make a summary statement about what students have learned (i.e., to report a grade or a standardized test score), that's called *summative assessment*. If the teacher's intention is to use the data to make decisions about next instructional steps, that's called *formative assessment.*

Formative assessment has been the focus of multiple educational studies and from which have resulted in a variety of definitions. For our purposes, we'll think about formative assessment as "encompassing all those activities undertaken by teachers, and/or by their students, which provide information to be used as feedback to modify the teaching and learning activities in which they are engaged" (Black & Wiliam, 1998 cited by Wiliam, 2011). Teachers have many decisions to make during a day, a month, a school year. Making good decisions about student learning often requires that teachers gather and analyze evidence to recognize students' current understanding. The analysis of student work (i.e., evidence or data) leads to decisions about what to include in Menu time, what models to show, what problems to design, whether to move on or reteach. Several researchers suggest we think of formative assessment as *assessment for learning.* (Fosnot, 2001) It's important to remember that formative assessment is a process that is continuous.

Additional Note:

We note that there are other forms of assessment that allow us to make different inferences (see Popham, 2018), that are neither summative nor formative in nature.

Screening assessments, while not yielding actionable information about specific student strengths and weaknesses, allow us to rapidly identify those students who are of concern and need further investigation.

Diagnostic assessments present a series of pre-determined probing, usually interview-based questions that help illuminate potential holes in student learning with respect to important milestones. (These milestones along learning progressions are discussed further in a later chapter).

If the teacher's intention is to make decisions about next instructional steps – what to include in Menu time, what models to show, what problems to set, whether to move on or reteach and with whom– that's called formative assessment. It "in<u>forms</u>" instruction. In this chapter we'll focus on some ways teachers can gather formative assessment in the math classroom to support student learning and reach *every* learner.

Samples of Formative Assessments

Students produce math work all the time. Worksheets, problem sets, homework, exit slips, etc. are a staple of any math class. They can also represent a rich, almost bottomless well of information about how students are thinking about the math they are doing. Teachers can tap into this well to gain a sense of what knowledge, strategies, and models students are bringing to a given model to make better instructional choices to move students to deeper understanding. In this section we will consider assessments, tasks, and work samples as well as anecdotal information that can be collected and analyzed when planning instruction.

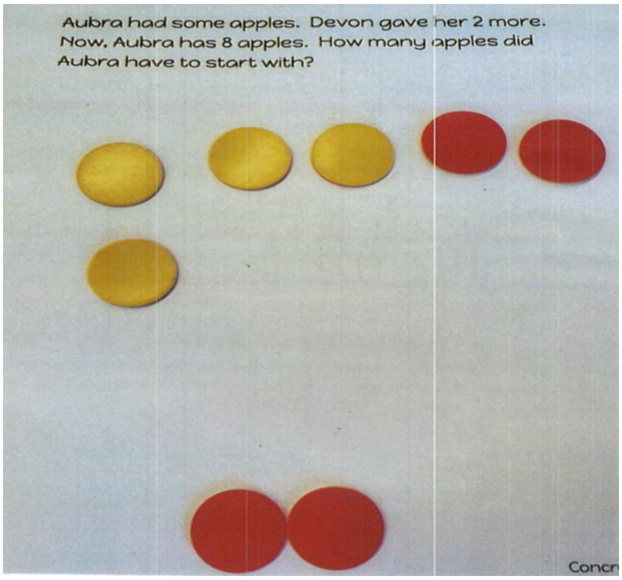
Concrete-Representation-Abstract (CRA)

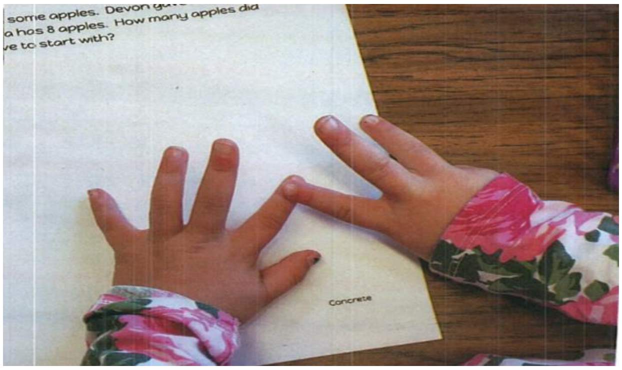
A CRA assessment provides an opportunity for students to solve a set of problems by demonstrating their thinking using three different models: a concrete, or physical model, a representational, or drawn model and an abstract model, or equation (Tapper, 2012). A CRA assessment is used with the whole class rather than just individual students. CRAs offer a way to get a look at everyone's thinking.

Each of these models highlights a student's developing understanding of specific mathematical concepts. The student work can be collected and analyzed to better understand a student's thinking and plan instruction accordingly. The students' work can highlight patterns in the use of models and/or strategies which might encourage a teacher to reflect on his or her instruction as well.

The following are a few examples of Grade 1 CRA tasks focused on additive reasoning strategies. The teacher recorded the student responses by taking pictures of the materials and methods used to solve the task. Many teachers position themselves at this "station" to snap pictures and conference with students when strategies are not clearly evident, or material/manipulatives have been moved. The representational and abstract tasks typically require only pencil and paper.

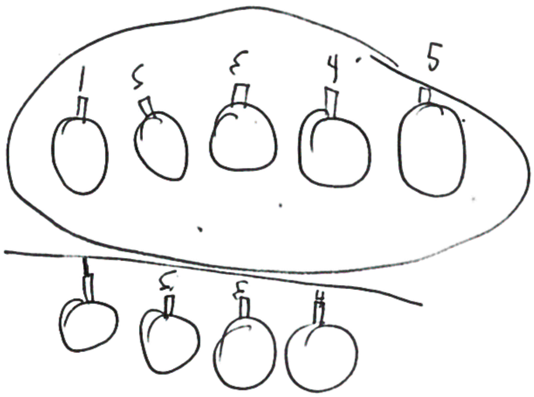
Concrete





Representational

Carson had some pumpkins. Natalie gave him 5 more. Now, Carson has 9 pumpkins. How many pumpkins did Carson have to start with?



Abstract

Kae'Mell found some leaves. Eva gave him 4 of the leaves she found. Now, Kae'Mell has 10 leaves. How many leaves did Kae'Mell have to start with?

$\boxed{6}+4=10$

Exit Tickets

Exit tickets are a formative assessment tool that give teachers a way to assess how well students understand the material they are learning in class. Teachers can decide when to use this type of assessment tool: daily or weekly, depending on what instructional decisions need to be made. A good exit ticket can tell whether students have a deep conceptual understanding of the content, or if misconceptions persist, or more time is needed for further development or practice of the concept. Teachers can then use students' responses for adapting instruction to meet students' needs the very next day. Some math programs even include exit tickets at the end of lessons or units.

Anecdotal Observations and Notes

Every day in every classroom, teachers observe students, whether it is during classroom instruction when students respond to content related questions or when students are having quiet moments of conversation unrelated to content. Many times we use these observations to inform our next instructional steps. We encourage teachers to become more deliberate and document these interactions in some manageable way with anecdotal notes.

Anecdotal notes do not have to be lengthy narratives. Many teachers use bullets, phrases or a quick summary written down in a notebook or on a clipboard that capture students' actions or verbal responses collected during individual conferences or quick check-ins during work time; whatever is manageable for them. These are factual observations that serve as evidence of a student's current understanding. It is important that the notes highlight students' strengths, in addition to misconceptions or errors, from which next instruction can be determined and linked to.

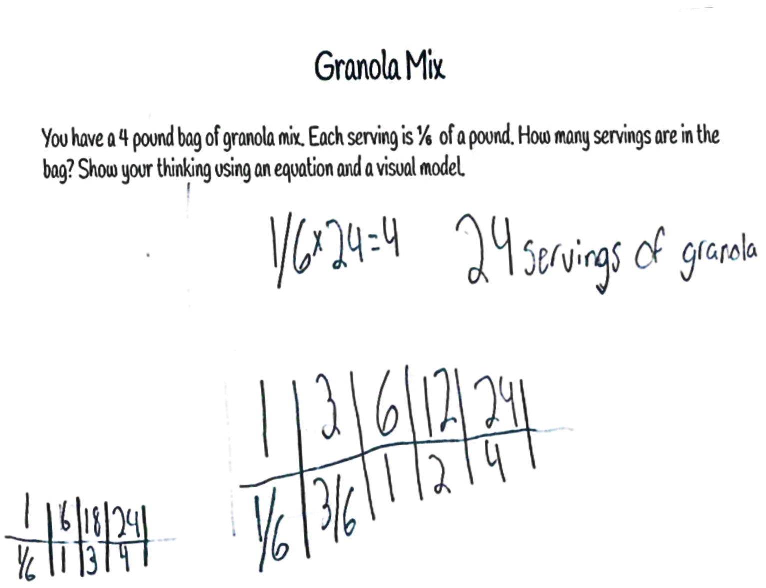
Table 1 Example of Anecdotal Notes based on observations of student work.

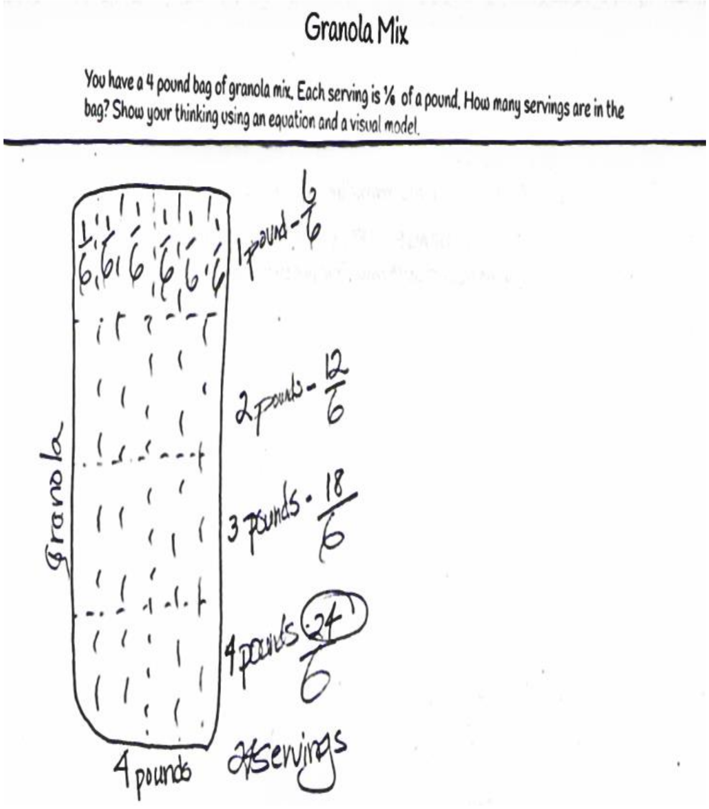
Do The Math A/B Menu Grab a handful Grab bag counting Which Sum Wins? Later add computer	Warm-Up ten frames flash and build	9/26 some counting to fill ten frames, did well with quick flash
	Build it quick Menu	9/27 Can say 5 on top of ten frame, but at times is still cf1 on ten frame Make 10 fish - use ten frames 7 and 4 confused fact Prompting him to make visual in his head
	Visual of ten frame - what does it look like? How many top/bottom? Empty?	

Work Samples

As we stated earlier, students produce work all the time. Teachers should feel obligated to collect and analyze student work regularly as a way to take stock of how students are making progress and understanding the mathematics they are engaging with. These work samples need to be more than just a page of computation practice. The work collected needs to provide evidence of student's use of strategies and models. Collecting student samples of solutions to problems allows us to look at student thinking at that time in order to create just-right instruction.

The following are work samples from a 5th grade task focused on fractions. What strengths do you see in the work and what instructional steps would you recommend next? These are the types of questions we should ask routinely when looking at student evidence of math understanding.





High Leverage Assessments

ALN has created High Leverage Assessments (HLA) for each grade level which are directly aligned to the High Leverage Concepts. The High Leverage Concepts, as talked about in the previous chapter, are a way to create *focus* on the most critical concepts to ensure that *all* students would have access to the opportunities that success in math facilitates. The High Leverage Assessments include tasks or problems that, when given to students, provide us with an opportunity to gain insight into students' understanding of the High Leverage Concept (HLC) for a specific grade level.

We will provide examples of the HLA tasks in the next part of this chapter as we share how we collect and analyze student work. We believe that the use of HLAs as formative assessment will enhance the creation of targeted learning opportunities that benefit all students.

Collecting Useful Student Evidence

We should be aware that not every question we pose or task we ask students to solve will provide us with useful information about next instructional steps. Consider the following questions: Solve 16 x 5 vs Use a model to solve 16 x 5. The first task can be solved simply and correctly by saying or writing 80. The second task requires students to create a model of their choosing to support their answer. Which task would offer teachers more information about next instructional steps? The obvious answer is the second task. These are the types of tasks we should use as tools for formative assessment.

Tasks given to students which only provide yes or no, correct or incorrect answers, afford teachers limited insight into students' thinking. Even with richer tasks teachers must look beyond correct or incorrect solutions to understanding students' strategies and thinking. Instruction needs to be tailored to meet all students' needs. There may be times when teachers want to check on correct and incorrect solutions, however we need to be clear about what the outcomes will tell us and how they will benefit students.

Analyzing and Sorting Student Work Using ALN Work Sort Protocol

If we can look more deeply at what a student is doing, look at their approach, strategy use and what conceptual understanding they are applying we can see where they have understanding from which to build and see where misconceptions or misunderstanding are evident. To illuminate "holes" or "gaps" in understanding, we can bring to the task of looking at any piece of student work a set of broad framing questions. These questions include:

- What is done successfully by the student?
- What strategies are being used to solve the problems?
- Is the problem being solved with manipulatives? … models? … by drawing? … using numbers or equations?
- What are the numbers in the problems?
- What knowledge and understanding is being applied to solve the problem?
- What pattern emerges from errors?

In other words, we can begin to get a sense of what the student is bringing to the work, in order to formulate next instructional steps that will support learning.

A protocol to sort student work to engage in the process of looking at what students can do and what they understand, regardless of correct or incorrect solution, has been drafted by ALN through the work of teachers, coaches, interventionists and facilitators. Using this protocol, or during any works sort, the important aspects to keep in mind are that the answer is less important than the thinking and work behind the solution, that understanding and what can be done by the student not only what they are missing is important to consider, and that noticing patterns and trends both as a class and for students across work samples will strengthen your instructional planning. We share this protocol with you below.

ALL LEARNERS NETWORK WORK SORT PROTOCOL

Best practice is to do this work with a team of colleagues.

Preparation

- Identify a task that will help illicit student strategies and evidence of student thinking to use in this protocol. The sources on alllearnersnetwork.com are:
    - High leverage assessments (HLAs)
    - Formative probes (CRAs)

- Consider the range of student strategies you are expecting to see on the task before administering the task to all students.

- Best practice is sorting with anonymity. Temporarily remove names by putting names on the back, post-its over names, etc.

Sorting and Analysis

- Do a first look through of work samples focusing on global noticings.
    - What's the good news? Look for and notice student strengths.
    - Consider the general overview of strategies, trends, and patterns.

- Next sort the student work by the students' strategies, regardless of the accuracy of their solution.

- Name each pile of student work based on the strategy used. Continue to look for patterns and trends in each pile and deepen the sort when necessary.
    - You will have a "?" pile for when you don't understand the student's thinking and follow-up is required.
    - When looking at a strategy pile, consider which students need to deepen their understanding of their current strategy and which students are ready to progress to a more efficient strategy.

Data to Action

- Record next instructional action steps based on insight from analyzing students' strategies and understanding. It is not the looking at student work that matters. It is what we do next in response to our formative assessment data that matters. Here are possible areas for your next action steps:
        - Flexible groupings for targeted instruction
        - Student follow-up interviews
        - Math menu options
        - Launch activities (number sense routines)
        - Entrance and exit tickets
        - New main lessons or reteach lessons
        - Select and sequence for strategy shares

ALL LEARNERS NETWORK
Math for Every Student

As teachers analyze and group work samples together based on similar strategies or approaches to a task, patterns emerge about what a student or group of students understand and are able to do. This analysis helps us decide and deliver their just right next learning opportunity. We can identify a variety of strategies students bring to problems. Often, a student will use a more advanced strategy in familiar situations but revert to less sophisticated strategies when the context is less familiar. These may include a variety of tools, manipulatives, representation, models, and strategies including: five and ten frames, number paths, number lines, place value models, fingers, bead racks, subitizing, counting, doubles, making 10 or 100, area models, decomposition, tape diagram, double number lines, graphs, tables and equations.

As work is sorted by strategy, teachers often end up with a pile of work that does not have enough information or clarity based on the evidence recorded to allow next steps to plan, collect these in a questions pile. The questions pile might require a quick check in with the student to clarify their work or a specific question, or it might result in follow up tasks or problems to gain sufficient information.

In general, the process is one of:

1. Gathering student work.
2. Sorting the work into meaningful categories.
3. Analyzing work in each category to determine where, along a learning progression, students are working.
4. Determining next instructional steps that will help the student solidify current understanding and move the student along a learning progression.

This sorting process provides actionable next steps for instruction based on evidence and conceptual development.

Use trajectories and frameworks to orchestrate target instruction

Once a teacher, or preferably a group of colleagues, has sorted student work by strategy they should collaborate to plan next steps for the class, small groups and individuals. The strategies and understandings that students use can be built upon through tasks, small group lessons and menu activities. Errors and misconceptions can be explored by students through targeted questions, problems, or activities presented by teachers.

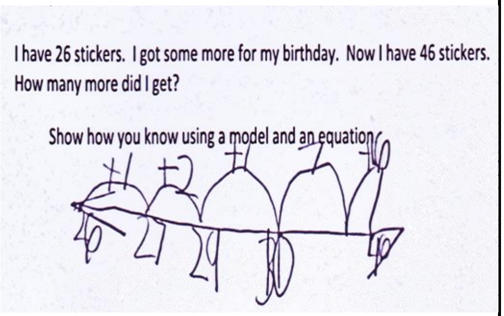
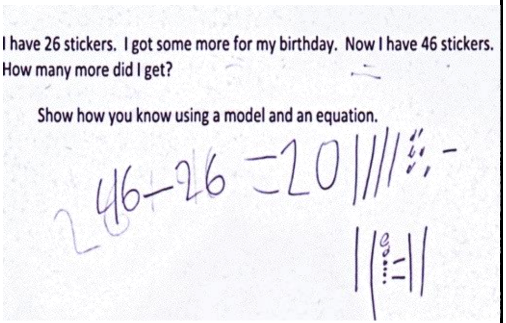
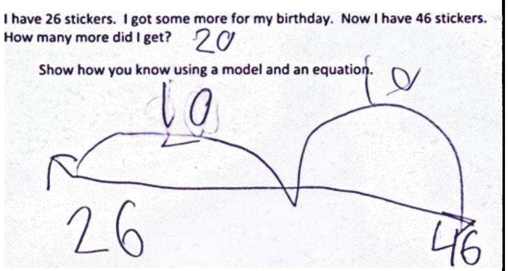
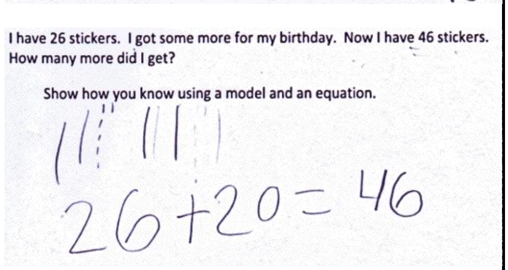
When a misconception or confusion or error pattern occurs for the majority of the class, that is an indication that the main lesson or launch portion of instruction needs to revisit or spend more time on the concept. Student work might show that your class needs more opportunity to develop additive strategies or spend more time exploring the use of ratios. For example, if most students in a class think that 7=7 is not a true statement, it would suggest incorporating more exploration of equality, possibly through lessons including balances, as well as exposure to equations in multiple formats.

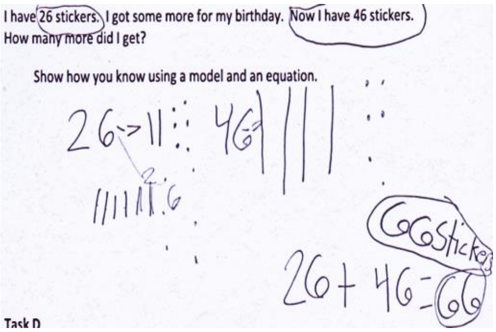
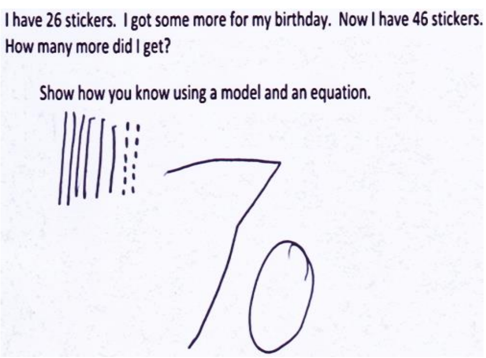
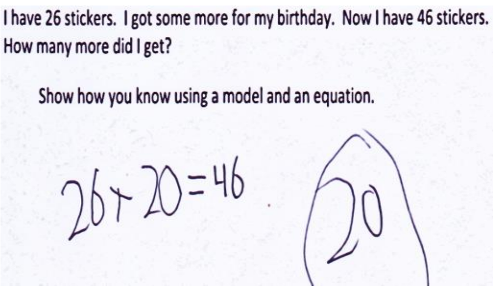
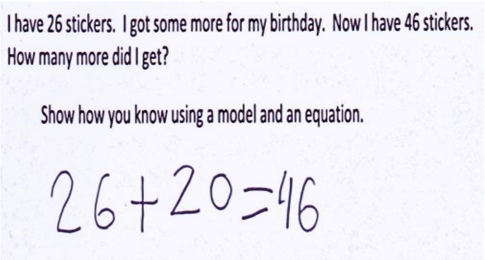
When you have groups of students with similar strategies, understandings and use of tools a teacher can consider what is the next step for that group of students - what step would you take to move their understanding along the trajectory. Using evidence from student thinking, teachers can provide "just right" math opportunities for their students. Through the use of intentionally selected problems, activities and games, teachers can address individual needs during Menu. Students are doing math at their personal instructional level, moving along a conceptual learning trajectory. This means that the content provided during Menu, as a result of formative assessment, might not always match the current grade level concept being studied during the Main Lesson. Menu also offers a time in the day, where students are engaged and working individually or with partners, when a teacher could pull a small group if the appropriate next instructional move included a small group with a teacher. It might be a group to revisit a problem, to discuss how to record thinking, to explore a concept further. Menu is used to provide opportunities to support mastery of concepts students have not fully developed. These are identified in the High Leverage Concepts.

To support the use of High Leverage Concepts, there are HLC Learning Progression documents for each HLC pre-K to 5. (See Appendix A for samples of the HLC Learning Progressions or on the ALN website here.) These HLC learning progressions provide a sequence of how skills generally develop in students - they are not a hard and fast timeline but they support planning the next "just right" step for a student to move their understanding along the continuum to master understanding of the HLC. Viewing student work in a broader context --- that of *learning progressions* --- can often reveal "holes" in a student's prior learning that are preventing successful application of the lesson at hand. Knowing how students think about the variety of mathematical contexts with which they engage can tell us what next steps might be needed to move student learning forward. Considerable work has been done researching and documenting mathematical learning progressions (*i.e. Clements & Sarama 2012,2014; Daro 2011; Maloney, et.al. 2014; OGAP frameworks, etc.*). These resources have been essential in the creation of the ALN learning progressions.

This table shows a completed student work sort from Task C on the grade 1 HLA. You will see work samples grouped together with notes around the work and strategy use as well as plans for next steps.

Table 2 Organization and Analysis Results from Student Work sort using ALN Protocol.

Student Work	**Strategy/Commonality**	**Next Steps**
	Addition Number Line - by 1s	Ask about answer and work on showing answer. Consider other models? Check Number line understanding.
	Found the difference Equation	Models how to represent more accurately.
 	Addition with place value Number Line Base ten	Ask about equation and answer and work on this as next steps as needed.

Student Work	Strategy/Commonality	Next Steps
 I have 26 stickers. I got some more for my birthday. Now I have 46 stickers. How many more did I get? Show how you know using a model and an equation. 26→11 46 66 stickers 26+46=66 Task D  I have 26 stickers. I got some more for my birthday. Now I have 46 stickers. How many more did I get? Show how you know using a model and an equation. 70	Had strategy to add w/ models	Needs to work on understanding problem. Retelling and modeling situations without questions or solving. Use problem introduction protocol.
 I have 26 stickers. I got some more for my birthday. Now I have 46 stickers. How many more did I get? Show how you know using a model and an equation. 26+20=46 20  I have 26 stickers. I got some more for my birthday. Now I have 46 stickers. How many more did I get? Show how you know using a model and an equation. 26+20=46	Solved with equation Clear answer Answer not clear, ask about that	Ask to do model in addition to equation and provide instruction on that if not able to.

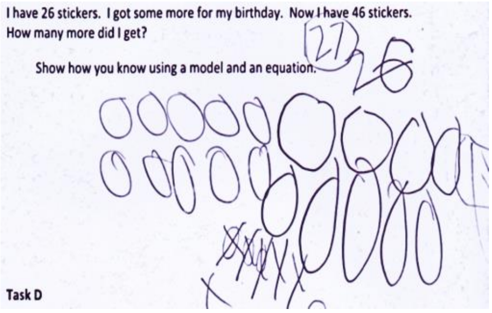
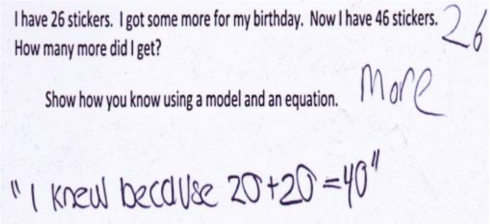
 I have 26 stickers. I got some more for my birthday. Now I have 46 stickers. How many more did I get? Show how you know using a model and an equation. 27 26 Task D  I have 26 stickers. I got some more for my birthday. Now I have 46 stickers. How many more did I get? Show how you know using a model and an equation. 26 More "I knew because 20+20=40"	What??? Need more info	Tell me about how you solved this problem. How did you figure this out? How does this help you answer this question? What is your answer?

Table 3 Organization and Analysis Results from data collected from K HLA

<u>Math Focus:</u> 1-1 Counting (up to 20 objects)

Student Stations

- Counting Collections - bags or collections given to students to count
- Grab Bag - Students reach into a bag/collection pull out a handful, count and record items
- Creations, Shape Puzzles or Lines - students build a creation or cover a shape/line with cubes and then count how many cubes used

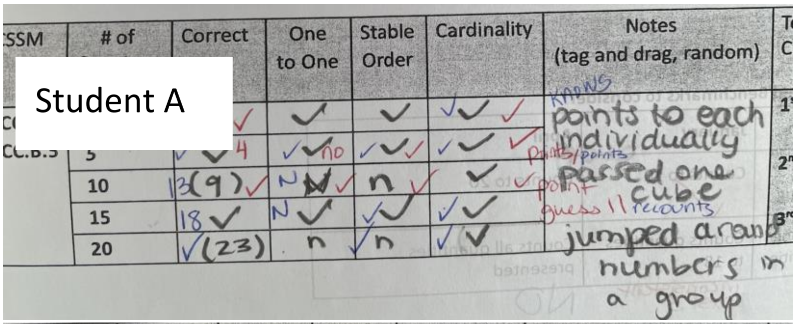
Teacher Station

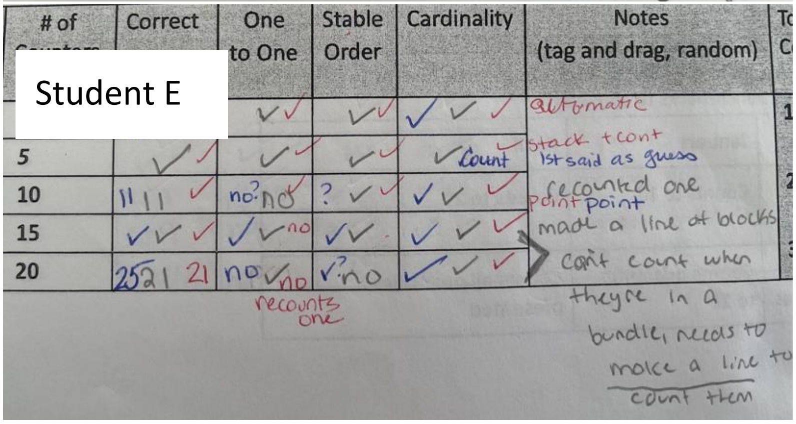
- Count and Dump - count collection into bowl/cup, dump and recount collection (Bridges Intervention)
- Making Towers - say a number, students build, (<u>Developing Number concepts Counting, Comparing Pattern</u> Kathy Richardson)
- Counting story - build with cubes, write numerals (<u>Developing Number concepts Counting, Comparing Pattern</u> Kathy Richardson)
- Hunt for it - items covered in tubs can either (<u>Developing Number concepts Counting, Comparing Pattern</u> Kathy Richardson)

***Independent counters don't need this focus for teacher station.*

Within 10

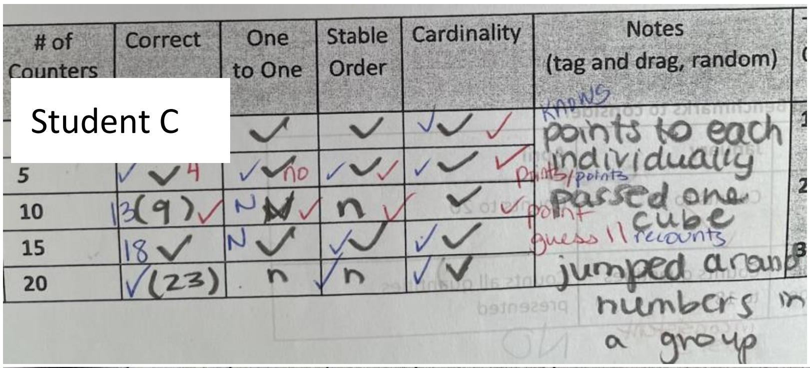
- need teacher check in 3-5x per week to teach around 1-1 either at a station activity or as a teacher station

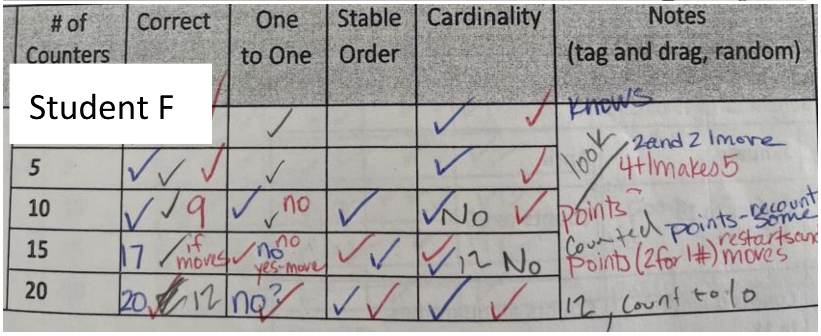




Within 10-20

- need teacher check in 2-3x per week to teach around 1-1 either at a station activity or as a teacher station





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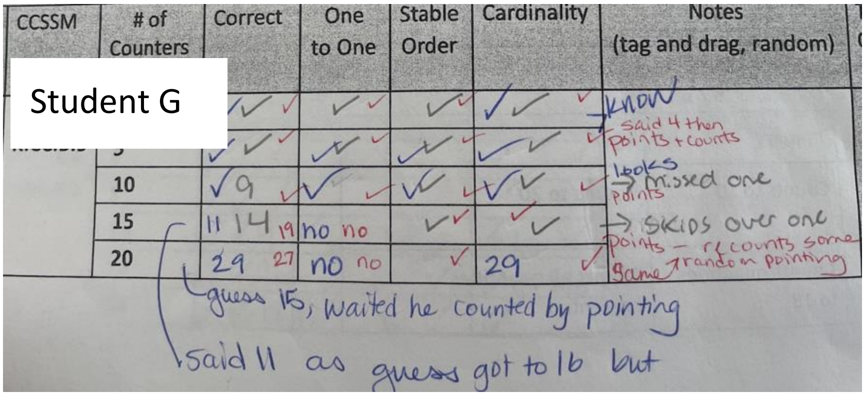
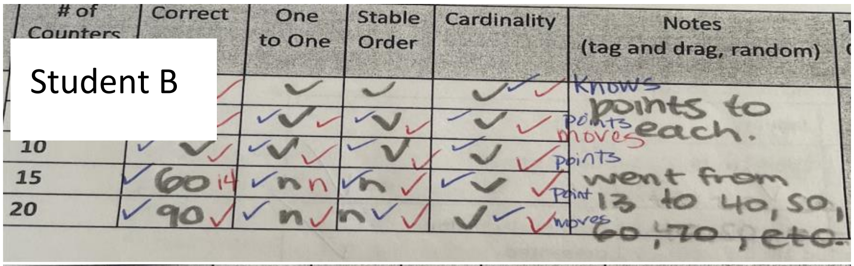
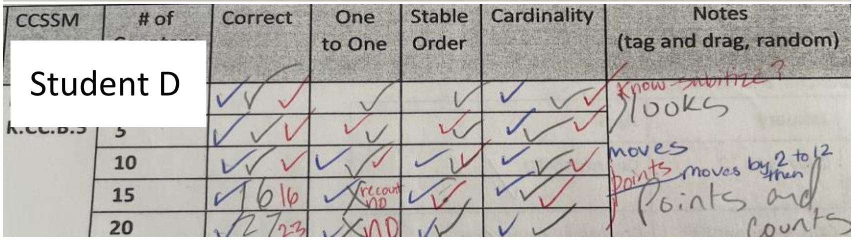
<tr><td></td><td>
</td></tr>
<tr><td>To 20 or beyond **
Independent at station
- had 1-1 can do larger collections, include one more/one less</td><td>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Working with anecdotes and observations can prove to be just as valuable a tool for planning as student work samples. For example, a teacher noted that some students were fluently rote counting by 10s but were counting by 1s when asked to add ten to a number. Based on observations of their work it appeared these students were not making the connection between counting by 10 and adding 10 to a number. This group of students was put in a small group where they completed a few tasks as a group. First ten frames were used to represent counting by 10s and the action of putting down another ten or adding ten to what was on the table was emphasized. Stopping at different points to discuss then ask “how many are on the table? Now if I add another 10 how many do you think there will be?” A pause from the teacher allowed students to think and consider what would be next, then the physical addition of the ten frame to the collection allowed them to check and verify as they began to make a mental model and see the connection between counting by 10 and adding ten. To allow for exploration of this concept in Menu, the students were taught Common Card Compare. Instead of Compare where you see who has most or least, you have a common card that is added to the card each player flips. In this case it was a 10 in the middle that had to be added to the card each player flipped over before comparing quantities. The game was taught, using ten frame cards so students had the visual scaffold and at the start some would still count to check their thinking, but it also allowed them to revisit and practice this skill and have the opportunity to recognize and discover the pattern. This exploration was also incorporated into the launch portion of the lesson. First it involved counting by 10s as continued practice but also number strings such as 12+10, 22+10, 32+10 where students were able to explore what happens when you are adding ten to a number. Through these explorations, students were able to discover that their rote counting by 10 patterns they had memorized was connected and then understand it was actually adding ten to each successive number.

After menus have been implemented, and to responsively plan classroom instruction, it is essential to progress monitor. Every week, or two weeks at the least, you need to follow up to determine progress that students are making with the concept focused during intervention and/or menu time. Then update the menu or interventions to reflect current needs. Without frequent monitoring, students could either spend too long on a concept which they have already mastered, or it would not be known that the approach tried isn't effective with that student and you need to try something else. Sorting work and completing regular formative assessments allow teachers to implement meaningful menus to support students in developing critical concepts. Teachers often have a lot of data that is collected and feel pressure to have students reach the end of year benchmarks. However, taking the time to look at where students current understanding is and to meet students where they are at can benefit the overall goal to meet end of year benchmarks. Taking the opportunities to look at progress along the way will allow movement to be recognized and aid in planning next steps to move learning forward.

Chapter Summary

1. We need to determine what skills, knowledge, and understanding students are currently bringing to problems in order to support their continuing progress in learning important mathematics.
2. The way in which we uncover student understanding is through routine use of formative assessment.
3. Formative assessment and the use of developmental learning trajectories are the foundation for planning intentional instructional decisions that benefit all students.

References

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Appendix A - Samples of the High Leverage Concepts Learning Progression for Grade 2 and Grade 4

(All PreK- Grade 5 HLC Learning Progressions can be found on: https://www.alllearnersnetwork.com)

Grade 2

Additive Reasoning
HLC: Use place value understanding to add and subtract numbers accurately, flexibly, efficiently, and strategically within 1,000 (in context and in equations) (NO standard algorithm)
September → **Grade Two HLC Learning Progressions** →June

Rote Oral Count Sequence *(rote counting from 1; rote counting from any start number)*

Counts FWD and BWD within 120 starting at any number	Counts FWD and BWD within 220 starting at any number	Counts FWD and BWD within 500 starting at any number	Counts FWD and BWD within 1000 starting at any number
Skip counts FWD and BWD by 10s starting at any number within 120 and on decade within 1000	Skip counts FWD and BWD by 10s starting at any number within 220	Skip counts FWD and BWD by 10s starting at any number within 500	Skip counts FWD and BWD by 10s starting at any number within 1000
	Skip counts FWD and BWD by 100s starting at century within 1000	Skip counts FWD and BWD by 100s starting at any number within 1000	

Place Value *(Students must use models to build understanding along this trajectory. Models should support students developing understanding of the magnitude of digits in their place values.)*

Tells the number one more and one less from any number within 120.	Tells the number one more and one less within 220.	Tells the number one more and one less within 500 starting at any number.	Tells the number one more and one less within 1000 starting at any number
Tells the number 10 more and 10 less from any number within 120	Tells the number 10 more and 10 less from any number within 220	Tells the number 10 more and 10 less from any number within 500	Tells the number 10 more and 10 less from any number within 1000
	Tells the number 100 more and 100 less from any century within 1000.	Tells the number 100 more and 100 less from any number within 500	Tells the number 100 more and 100 less from any number within 1000
Uses place value understanding to compare any 2-digit numbers to each other.		Uses place value understanding to compare any 2-or 3-digit number to each other.	

Using Place Value for Addition and Subtraction *(Students must use models to build understanding along this trajectory. Models should support students developing understanding of the magnitude of digits in their place values.)*

Tells the value of the digits in any 2-digit number. Decompose any 2-digit number into its place value parts.		Tells the value of the digits in any 3-digit number. Decompose any 3-digit number into its place value parts.	
Uses understanding of combinations to 10 to find combinations to 20.	Uses understanding of combinations to 10 to find multiple of 10s partners to 100. (e.g. 6 + 4 = 10 so 60 + 40 = 100)	Uses understanding of combinations to 10 to find multiple of 100s partners to 1000. (e.g. 6 + 4 = 10 so 600 + 400 = 1000)*** Also, 560 + 140 = (500 + 100) + (60 + 40) = 600 + 100 = 700	Uses understanding of combinations to 1s, 10s, 100s to add any numbers within 1000. (e.g. 538 + 212 = (500 + 200) + (30 + 10) + (8 + 2) or 538+2=540 540+10=550 550+200=738

*For various quantities, students may compare by subitizing, matching (1:1) lining items up, or counting quantities
This concept is also impacted by conservation of number - consistent count regardless of orientation ("It is still 4, the cubes are just spread out"*)

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Grade 4

Multiplicative Reasoning				
HLC: Multiply and divide within 1000 within context and with equations.				
September →	**Grade Four HLC Learning Progressions**			→June
Students must use models to build understanding along this trajectory. Models should support students ability to unitize- understand a group or collection of items represents 1. For example, one group of 5 consists of 5 individual items but is classified as one group.				
***Students are maintaining and using their fact strategies to solve basic facts through 100*				
Extending Single Digit Fact Strategies to Greater Numbers (Partial Products and the models that go with it) **Deriving strategies through area model and decomposition of numbers through use of understanding of facts**				
Uses the area model to decompose into smaller arrays- like a 24 x 6 array is also a 20 x 6 + a 4 x 6 array.	Uses the area model for products to 1000 (100 x 100) to understand length and width as dimensions that are 1 x 1 square units. Count by ones or skip count rows or columns.	Decomposes of side lengths to use the distributive property with numbers through 100 x 100.	Uses the partial products method to solve multiplication problems with numbers through 100 x 100	Comparison by use of factors.
Division (Partitive and quotative models) in both equations and contexts (Using area models, number knowledge, or sharing models)				
Subtracts equal sized groups from the total by repeatedly removing (sharing) benchmark sized parts (10, 5 and then 1s) from a whole within 1,000. AND/OR Subtracts equal sized groups from the total by repeatedly removing the divisor.	Uses partial quotients, and repeatedly removes similar sized products using the divisor as a factor. Uses inverse relationship, and consider the missing factor problem for multiplication to solve a division problem.			Uses partial quotients, removes larger-sized products using the divisor as a factor, multiples of benchmark numbers, and multiplication facts.
Place Value *(Students must use models to build understanding along this trajectory. Models should support students developing understanding of the magnitude of digits in their place values.)*				
Understands expanded notation of numbers to 1000. Students are thinking multiplicatively (e.g. 325 is (3x100) + (2x10) + (5x1) AND Understands that each place is 10x more as you move to the right.		Uses place value understanding to multiply single digit times 10 and extend understanding of single digit x single digit to single digit x a group of ten (e.g.- size of unit - 3 x 1-one extends to 3 x1-ten).	Uses place value understanding to decompose factors to multiply using area model and partial products.	Uses place value understanding to multiply a single digit by multiple of 10 and extend understanding of single digit x single digit to single digit x multiple of ten (e.g. 3 x 6 ones extends to 3 x 60 or 3 x 6-tens).