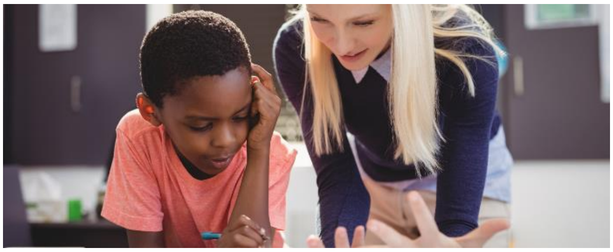

What's Important? High Leverage Concepts

CHAPTER 3

What are the HLCs?

We wanted to create *focus* on the most critical concepts to ensure that *all* students would have access to the opportunities that success in math facilitates. At each grade level we asked the questions: *Which concepts, or big ideas, have the most leverage for future success? What would a student have to understand in one grade to be successful in the next?*

Why High Leverage Standards?

Success in mathematics is a big deal. It's linked to high school graduation, and to college and career readiness (Chaudry, 2015; Laby, West & Voloch, 2015; McClarty, 2016). For that reason, mathematics is a gateway subject. Early success in mathematics means a higher probability that students will achieve in secondary and post-secondary education (Galindo & Sonnenschein, 2015). To provide equity to students, and to close existing achievement gaps with students of color, students from poverty, and students with IEPs, all students need to know some mathematics, particularly algebra (Snipes, Finkelstein, Regional Educational Laboratory West (ED) & WestEd, 2015). Math isn't just a subject for students headed for STEM careers. Even for those in nontechnical fields, it's evidence of readiness for college and beyond.

Not all the math identified in the Common Core State Standards (CCSS), however, leads to algebra. Some math is more critical to learning algebra and the potential opportunities that comes with it. In particular, what the National Council of Teachers of Mathematics (NCTM) refers to as *numbers and operations*, is often seen as the essential math content to lead to algebra and beyond (need others, Wilson, 2009).

When we created the High Leverage Concepts (HLCs) we used these understandings to guide us. We wanted to create *focus* on the most critical concepts to ensure that *all* students would have access to the opportunities that success in math facilitates. At each grade level we asked the questions: *Which concepts, or big ideas, have the most leverage for future success? What would a student have to understand in one grade to be successful in the next?*

The HLCs, and the MAPS – diagrams that connect instructional information relevant to each HLC – were created to provide a *continuity* from one grade to the next. The ideas that flow in the HLCs are a learning trajectory that leads, ultimately, to algebra. For students who have learning challenges and/or struggles with mathematics, the HLCs can provide a clear path for intervention and a focus for effort.

Why is an emphasis on focus and continuity so important? These are two elements that create real opportunities for equity, because these concepts help keep students' and teachers' attention on what's most important. An analogy from informal learning is helpful here.

Imagine a baseball program for children where some players were only allowed to wear a team's uniform, but they were never allowed to play. In this scenario, the majority of a team's players would practice together and play games. But a small minority would only be allowed to watch - occasionally. Most of their baseball time would be spent on a separate field, with a different coach (who never spoke with the team's manager) practicing discrete skills. These few "special" players would spend their time swinging the bat, running around the bases, and playing catch. They would never have the opportunity to play in a game or understand how the skills they were learning fit into real baseball.

This is a strong analogy for the experience of students with learning differences when it comes to learning mathematics. They don't get opportunities to "play a real game", to do real mathematics. When we introduce HLCs into this experience everyone -- the teacher, the special educator or interventionist, and the student –- knows what the important goals are. There is also a clear path from one concept to the next. In this context, learning math in the general education classroom becomes a bit more straightforward. By focusing more on what matters most, we create opportunities for all learners to "play." This is a chance for students, even students with learning challenges, to do "real math" (problem solving, applied tasks) rather than spending all their instructional time on discrete skills.

What are HLCs?

The HLCs are all about focus and continuity. The HLCs are a tool to facilitate focus and continuity by providing clear guidelines about what matters most, and effective tools for teaching these concepts.

An HLC is broader than a standard, like those in the CCSS, although they are connected. In general, though, HLCs are aimed at deep conceptual understanding of key mathematical concepts. They imply that a student can *demonstrate* their understanding using a conceptual model. For example, the HLC for Grade 1 states:

Understanding of number values and sequences to 120 (cross century, cross decade)

Understanding place value when adding and subtracting numbers within 100 (in context and in equations)

The big idea for first grade is for students to deeply understand additive reasoning with whole numbers within 120. This is the "elevator speech" for first grade about what's most important for *everyone* to understand. It's the minimum expectations for every child leaving first grade. The *minimum*. By drawing a line in the sand around a particular essential concept, teachers can ensure that there is equity as students move to second grade.

But aren't *all* the concepts in the Grade 1 curriculum important?

A comprehensive curriculum *is* important. A comprehensive curriculum is built as though all math concepts and understandings at each grade level are equally important. We know that they are not. Algebra, for example, is a good predictor of school success. Understandings that lead to algebra will create more opportunities for students as they progress through the grades. In the long term those understandings are more important.

Consider the geometry strand in most standards. For illustration, we can look at the CCSS for Grade 1 in geometry. These standards are built around the idea that students will "*Reason with shapes and their attributes."* No one would argue that this is not a worthwhile area of study. However, if a student were unable to demonstrate reasoning with shapes, would it interfere with their success in second grade? Would it keep a child from being able to access algebra later in their career? Contrast the importance of *reasoning with shapes* to *using place value when adding and subtracting numbers within 100*. Not being able to add and subtract, and to understand place value will have a much greater impact on math learning in second grade, than an understanding of shapes. Both are important math concepts for children to be exposed to and, hopefully, to learn. There are some concepts in the comprehensive curriculum, though, which have more *leverage*. That is, they are more important because of their long-term impact. This is the point of the High Leverage Concepts

Why not just teach the HLCs? Because the HLCs at each grade level are *minimum* understandings. They are like saying, "If a child only really understands one or two things at a grade level, what should those things be?" The HLCs are not, and do not profess to be a curriculum. The CCSS offers a comprehensive approach to specific, testable math understandings. Curriculums based on standards like these are important for students to develop a complete understanding of mathematics.

The Role of HLCs in Intervention

Identifying the most important math content is especially helpful when trying to understand how children should be supported when they are "behind" their peers. For these students, the most important concepts are the ones that will help them most in their efforts to do grade level work with peers.

For this reason, both special educators and math interventionists have found the HLCs particularly helpful in their planning. Many special educators who use the HLCs find that they can be used as a guide when planning yearly IEP goals. Since the HLCs provide a continuum of concepts, they can also be used to backward plan, using early concepts as steppingstones to the required grade level concepts students must report on their individual programs.

The MAPs

The High Leverage Concepts are organized into Mathematical Archical Progressions (MAPs) to make using the information in the HLCs easier to use and to connect assessment and instruction. These deal with a developmental progression of the "archical" (primary, chief) mathematical concepts in school. The meaning of the acronym is something we seldom need. People who use the HLC MAPs tend to think of them like a roadmap: They offer the user a chance to know a little more about where they are and where they're going.

During the creation of the MAPs, our goal was to try to put enough information together so that classroom teachers could find a simple reference for the highest leverage concepts at their grade, and surrounding grades, along with guidance on key elements in those concepts. These elements include models that have been found to be effective with general education classrooms, as well as models that are particularly important (and effective) for learners who are struggling. Finally, the MAP identifies strategies that are critically connected to concept understanding. That is, students who can demonstrate deep understanding of the HLCs often rely on these strategies.

Here are the PreK-Grade 2 MAP and the Grade 3-5 MAP. The HLCs are at the heart of all ALN professional development focused on mathematics instruction.

ALL LEARNERS NETWORK
Math for Every Student

High Leverage Concepts Pre-K to Grade 2

Number		Additive Reasoning	
Pre-K (4-5-years old)	**Kindergarten**	**Grade One**	**Grade Two**
Understanding of number values and sequences to 10 (counting, cardinality, conservation and stable order) 1:1 Correspondence	Understanding of number values and sequences to 20 (counting, cardinality, and stable order) 1:1 Correspondence Comparing quantities	Understanding of number values and sequences to 120 (cross century, cross decade) Understanding place value when adding and subtracting numbers within 100 (in context and in equations)	Use place value understanding to add and subtract numbers accurately, flexibly, efficiently, and strategically within 1,000 (in context and in equations) (NO standard algorithm)
Models for Intervention			
5- and 10-frames for counting	10 frames	Multiple 10 frames	Multiple 10 frames/strips Number lines/adding up (using part-part-total focus). These *start from zero*
Models for Instruction			
Fingers 5-frames for subitizing Dot patterns (*regular and irregular*)	Fingers Dot patterns (*regular and irregular*) Bead Racks	Number - number paths Adding and subtracting - Multiple 10 frames, ten strips	Place value materials (e.g., base ten blocks/pieces, digi-blocks, 10 frames, 10 strips) Number lines (articulated)
Critical Strategies			
Counting by 1's, Subitizing, Organizing[1], Tracking[2]		Using Doubles, Making 10, Making 100, Counting by 10's and 100's (start on any number) – moving forward and backwards	
Number		**Additive Reasoning**	

ALL LEARNERS NETWORK
Math for Every Student

High Leverage Concepts
Grades 3-5

MULTIPLICATIVE REASONING		FRACTIONS
Grade Three	**Grade Four**	**Grade Five**
Multiply and divide numbers within 100 (in context and in equations)	Multiply and divide any two numbers within 1,000 (in context and in equations)	All four operations with fractions (in context and in equations (NO standard algorithms – using modeling and/or decomposition approaches.
Models for Intervention		
Strong connections between grouping and area models. Use of area models for multiplication facts.	Area models to support decomposition for multiplication. Partitive (sharing) models for division.	Area models to build equivalence for add/sub. Parts/whole models for multiplication, with a focus on whole numbers x fractions. Measurement models for division of fractions.
Models for Instruction		
Grouping models (i.e. circles and stars, loops and groups, beans and cups), jumps on a number line, repeated addition, skip counting; area models for products to 100 (may start by using place value blocks) Experience with both partitive (sharing) models and quotative (partial quotients) models	Area models for products OR quotients to 1,000 The use of area models to develop decomposition strategies for multidigit computation Experience with both partitive (sharing) models and quotative (partial quotients) models both in equations and in context.	Area models for part/whole relationships, place value blocks, Cuizenaire rods, fraction bars, fraction pieces, geoboards, pattern blocks
Critical Strategies		
Decompose multiplication expressions into 2's, 5's, 10's Model Division as sharing among groups OR computing how many groups of a specific size are in the whole: 6÷2 can be 6 shared between 2 groups (3 in each group) OR 3 groups of 2 in 6 (2+2+2 or 3x2)		Identifying equivalents for benchmark fractions Decomposing benchmark fractions (and 1 whole) Showing position of benchmark fractions on the number line

Quantitative Reasoning strands

At the top of each MAP, quantitative reasoning strands are identified. These include, *Number, Additive Reasoning, Multiplicative Reasoning,* and *Fractions.* These are the broad cognitive topics that guide essential instruction at a given grade level. In the early grades, for example, students are focused on developing basic number concepts.

These are important foundations that occupy students' thinking about math from birth to about age six. Again, this is not to suggest that children at that age are not engaged in *lots* of math thinking. They're recognizing patterns, shapes, and organizing important ideas like magnitude and subitizing small collections of objects. By creating a quantitative reasoning theme, *Number*, we are acknowledging that the equity focus of these years is about number values, correct sequencing, and one-to-one correspondence.

Why worry about this type of reasoning focus? Various researchers over the last 50 years have determined that quantitative reasoning is *progressive* and *hierarchical*. Quantitative reasoning is progressive, because concepts that are learned in additive reasoning lead to multiplicative reasoning and then to fractions (and proportional reasoning). These represent *stages* in the development of number concepts. The idea that reasoning is progressive suggests that students will move through stages in a fairly regular order and at a (more or less) predictable rate. The progression of quantitative reasoning that is captured in the HLCs also tells us that students need to have some facility with early stages if they are to make sense of later ones. A student who cannot demonstrate understanding of Number, for example, will have a hard time making sense of the part/part/whole relationships embodied in additive reasoning. Likewise, a student who does not have an understanding of the joining or correspondence (unitizing) of groups will have a difficult path to understanding multiplication.

The quantitative reasoning captured in the HLCs is also *hierarchical*. That is, the upper level concepts contain the elements of early concepts and add something more. Number concepts are included in additive reasoning, but additive reasoning adds additional cognitive demand, and the development of more nuanced and sophisticated understandings. The hierarchical nature of quantitative reasoning in the HLCs is an acknowledgement that, as students progress, they are not just *learning more*. They are *learning to think differently*. We do not simply suggest, for example, that students know "how to add and subtract." Adding and subtracting can be (and often are) taught to students as a series of steps to be carried out without understanding. We include the ability to add and subtract as part of a deeper understanding of place value in the HLCs under Additive Reasoning because they are evidence of learners' new reasoning abilities, the growth of more sophisticated ways of understanding numbers. A student who demonstrates understanding of HLC 1 must show the growth of additive reasoning by showing the part/part/whole relationship that additive reasoning is built upon.

Likewise, multiplicative reasoning extends the notion of *joining* and *separating,* that are essential to additive reasoning, in new ways. In multiplicative reasoning, students must join *groups* of things. The number of items in each of these groups must be the same. The process of counting using groups is called, *unitizing*. This idea is built on the joining and separating of additive reasoning, but now using groups. In this way the numerical operations of addition and subtraction are transformed into the operations of multiplication and division.

By the time we get to fractions in fifth grade, students have built a strong ability to reason with numbers both additively and multiplicatively. Fractions are, perhaps, the clearest representation of the progressive and hierarchical nature of quantitative reasoning in the HLCs. Fractions do not offer new operations. There are not new ways of reasoning with numbers. What changes in the student of fractions is the nature of the numbers themselves. All the reasoning that applies to adding, subtracting, multiplying, and dividing whole numbers applies to fractions. This is why the progression of the HLCs creates a strong focus on reasoning with whole numbers before attempting deep work with fractions. Everything students have learned about operating on whole numbers can be applied to operating on fractions. The thoughtful connections of previous understanding to new learning (making use of progression – that which has led up to fractions, and hierarchy – using everything learned for the new concept) make the learning of fractions straightforward. The lack of an approach which leverages whole number reasoning for learning fractions is why so many learners have difficulty with fractions. The result of those difficulties is often even greater problems with algebra. As we pointed out earlier, there are long lasting effects to failure with algebra.

Models for Instruction

Under the HLCs on the MAP is the row that includes *Models for Instruction*. These are a relatively new addition to the MAP and came at the request of teachers who were using it. At a gathering of teachers who used the HLCs over a period of at least a year, the suggestion was made to include instructional models for the general education classroom. The suggestion was to articulate specific models that would support the development of a wide variety of strategies in the general education classroom.

The use of models – for instruction or remediation – is a critical element of the High Leverage Standards. The deep understanding that the HLCs promote can only be achieved by *every* student if the instruction is guided by exploration of conceptual models. There are essentially two approaches to solving mathematical problems or tasks. One can manipulate models to create or construct a solution or one can follow an accepted procedure. Over the last 50 years or so the majority of instruction at all grades levels (though, perhaps less in the early grades) has focused on following accepted procedures to solve mathematical problems. Most of us have numerous, clear memories of being shown how to add or subtract, how to multiply and divide. After we were shown, we were asked to duplicate the teacher's method. If we were able to solve a given problem just the way the teacher did, we received praise and good grades. If we didn't do it that way, we were wrong. This approach has, arguably, led to the relatively weak mathematical understanding of American children (see TIMSS 2015, PISA 2018). It has also created an achievement gap for students of color, students living in poverty, and students with IEPs (Condition of Education 2016, NCES).

The alternative to trying to duplicate the teacher's thinking is to do our own. An approach that makes use of instructional inquiry – and the use of conceptual models – asks students to make meaning of mathematical ideas at a personal level. Instruction that is truly aimed at high leverage concepts must allow these understandings to develop for the learner her/himself. The use of instructional models is a way for teachers to facilitate the development of this understanding without resorting to telling students how a problem *should* be solved.

The advantages that inquiry instruction affords are better documented elsewhere (National Research Council, 1996; Universal Journal of Educational Research, 2014; Assessment & Evaluation in Higher Education, 2015). To understand the importance of models to learning of high leverage concepts, one must accept that they are the linchpin for successful understanding.

The list of instructional models that are offered in the MAPs is not exhaustive. There are MANY mathematical materials that serve as effective instructional models.

We've identified just a few of these. When choosing which models to include, we used three criteria:

Models we included should have multiple applications
Models we included should not have their own learning curve
Models we included should be inexpensive or free

Teachers implementing the HLCs found that some models were "recyclable," while others were not. Ten frames or number racks could be used, for example, for a wide variety of number applications beginning in kindergarten and running all the way through grades two or three. A hundreds chart, by contrast, while useful in some situations, is far less adaptable. It tends to be used in specific applications. Area models can be used from grade three through high school, in certain situations. It can be used to develop ideas in multiplicative reasoning, fractions, proportions, and polynomials. For that reason, it met the criteria of having multiple applications.

Numerous instructional models offer benefit to teachers and students, but at the cost of time to learn how to use them. Cuisenaire rods are an excellent resource. Yet, students must invest time in learning how to use the rods. There are activities, for example, that require students to learn about the relationship of the colors to their magnitude. For many teachers this is a minor drawback. However, the goal with HLCs is to be sure that EVERY student develops key understanding. National assessments reveal that a substantial number of students have only a *Basic* level of understanding of mathematics (National Assessment of Educational Progress, 2017 results). When time for learning is precious, we felt that the models used should be intuitive and easy to understand.

Finally, we want the models we chose to recommend for instruction leading to understanding of the HLCs to be inexpensive or free. This requirement was directly related to the belief that the focus and continuity that HLCs provide could help close achievement gaps. If the models we championed had associated costs, it would create another barrier to achievement for historically marginalized groups. For that reason we committed to including models that were free or relatively inexpensive.

Models for Intervention

The *Models for Intervention* row in the MAP is similar to the *Models for Instruction* row. It identifies important models to support student learning. The difference with these models is that they are used to help students who are having difficulty with understanding.

Early in the development of the HLCs, we wondered how to support students who were struggling with key concepts. At various points, after formative assessment (often using the HLAs) in the school year when some students were having great difficulty with grade-level HLCs, how could teachers help them? We set out to explore answers to this question with dozens of teachers, coaches, and interventionists across three states. The result of that inquiry is in *Models for Intervention.* These models are the ones that, according to our experience in the field, were found to be more successful with students who struggle. We are not suggesting that there are other approaches that might work or that these models will always work. All things being equal, if an interventionist or teacher is looking for a potential approach that will support a student who is having trouble understanding a concept, the model listed is a good way to go.

A good example can be found with the *Models for Intervention* for HLC 2. A large elementary school gave a midyear assessment to its second graders. After the assessment it was discovered that half the students (about 52%) could not subtract accurately within 1,000 either in context or in non-context computation. After some work with their instructional coach on the HLCs, some of the teachers elected to work with struggling students by investigating a number line model and "adding up" during subtraction. Instruction proceeded normally, but when students who were struggling received differentiated instruction on subtraction, they focused on the number line model and explored how to use adding to solve subtraction problems. At the end of the year, teachers who adopted this *Model for Intervention* to support learners saw over 90% of their students demonstrate understanding. While those who did not focus their instruction saw almost now additional gains.

Our knowledge about models for intervention is growing and expanding, particularly for students who have severe learning challenges in math. We hope that we can add new information to this part of the MAP as we gather more data from practitioners in the field.

Critical Strategies

Like *Models for Instruction*, *Critical Strategies* comes from the experience of dozens of teachers in the field, observing students and supporting understanding. We added these to the MAP because they identify particular strategies that demonstrate understanding, they offer a potential avenue for a student having trouble to connect important ideas, and they can offer instructional moves for teachers when they gather data from formative assessment.

Critical Strategies are one way for a teacher to formatively assess student understanding. One of the Grade five strategies, for example, is for students to demonstrate that they can decompose fractions. This means that they can take a number like 5/6 and know that it can be decomposed into:

1/6 + 1/6 + 1/6 + 1/6 + 1/6or

1/3 + 1/6 + 1/6 + 1/6 or

1/3 + 1/3 + 1/6 or

½ + 1/6 + 1/6 or

½ + 1/3

Why would this be such a critical strategy? Decomposition in this way demonstrates a deep understanding of equivalence, perhaps the most important concept in adding or subtracting fractions. If a student can demonstrate this strategy, it is a pretty good signal that he/she has developed an understanding of equivalence. If they can only decompose with sixths, equivalence may be an area of emerging understanding or misconceptions. Identifying critical strategies helps teachers to focus on areas of student work (both formal and informal) that offer evidence of understanding. This can be very helpful instructionally.

When, for example, first grade teachers collected samples of the HLA 1 in the fall of the school year, they did not expect students to have correct answers. It was, after all, the beginning of the year. The assessment *did*, however, yield some information on strategies that was useful for planning instruction. Teachers looked to see if students used direct modeling (tallies, for instance), jumped on a number line, or showed evidence of more sophisticated strategies. Teachers found that students who used a knowledge of doubles (or doubles + 1), or “made 10’s” were more efficient and often more accurate than those who used direct modeling. Instructionally, teachers (and coaches) used the information about strategies to plan activities for individuals and/or groups of students. The goal of developing the *Critical Strategies* of using doubles or making 10's provided a focus and helped teachers support their students to achieve understanding of HLC 1.

The "critical" nature of these strategies is supported by observation in the field by multiple practitioners, rather than from empirical verification. Still, the usefulness of these strategies for supporting student learning has been well-documented. Students who have difficulty with memorizing times tables, for example, can make good use of the strategy to learn multiples of 2, 5, and 10.

Consider a student who doesn't know their sevens times table trying to compute 4x7. This student could apply the strategy of using 2x7 and then doubling the result. If this same student wanted to compute 7x9, she could simply compute 7x10 and subtract 7. Using multiples of 2, 5, and 10, (and 1), it's possible to decompose any number for computation. Some special educators who have worked with this particular strategy have suggested that it could (should?) replace the many hours their clients with memory challenges spend trying to memorize multiplication facts.

Critical strategies, then offer teachers a way to assess student understanding. They offer a lens for viewing student thinking with an eye to moving them into deeper understanding. In some cases, these strategies can also provide interventionists and/or special educators with a tool to support students as they work toward understanding of the HLC.

The final section: Is there a best way to use this resource?

You COULD jump to your grade… but progressive, hierarchical means you should be familiar with all the grades on the HLC Maps, especially those that encompass the grade levels you teach. Take some time to read the entire HLC MAP that encompasses the grade level you teach. We believe you'll find the HLC MAPs invaluable in helping you craft targeted instruction for your students.