CHAPTER 1

What it Means to Teach for All Learners an Overview

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What does it mean to learn mathematics?

For many of us it meant memorizing things. We memorized math facts and procedures for adding and subtracting. We memorized the quadratic formula and the area of a circle. If we could remember all those formulas and facts, we did well on our math grades. Our parents were pleased. We were happy. But were we able to access the kind of thinking that mathematics brings to us? Had we developed the skills to solve complex problems? Did we develop the ability to analyze subtle patterns? Had we become comfortable with the process of discovery?

Certainly, if we have all that information about numbers at hand, we know *something* about math. Still, at least for Americans, a large percentage of us are functionally unable to use whatever information we were exposed to in high school and college. We see math as information that we don't really understand. It's as if we had memorized lots of dates and events but had almost no idea how they formed the story of history. We know *about* math, but we don't understand it.

That math has been taught and learned in much the same way for the last 30 years is puzzling. In 1989 the National Council of Teachers of Mathematics (NCTM, 2008) produced a new set of professional standards that should have refocused math instruction away from memorization and toward understanding. For some teachers, in some schools, instruction *has* changed, and as a result, their students learn math with understanding. For others, often schools that serve poor students and students of color, instruction is still focused on memorization, with less emphasis on understanding mathematics (Wenglinsky, 2000, 2001, 2002, 2004). The result of this poor instruction is that the opportunities that knowing math provides – access to half the majors in college, higher paying jobs, greater economic opportunity – are lost to students from historically marginalized groups. The disparity in math instruction, along with the challenges faced by students living in poverty, students of color, and students who are learning English, has created a math achievement gap that's lasted more than 50 years (Clotfelter, Ladd, Vigdor, Wheeler & Duke Univ., Durham, NC. Terry Sanford Inst. of Public Policy, 2006).

The goal of this book is to give you the tools to be the kind of math teacher that closes the achievement gap in mathematics, creating opportunity for all the students you teach. What we're presenting here is an approach to math instruction for *all learners.* This work has been informed by the design research[1] and collaboration among dozens of teachers and math coaches trying to actualize the ideal that every student can

1 Design research is a term that comes from engineering. Broadly speaking, it means trying out approaches to see what works.

learn math. This started as the All Learners Project and, after expanding to hundreds of classrooms is now called the All Learners Network.[2]

In this book, we'll first look at a method for teaching math that allows for learning grade-level math while also supporting remediation and scaffolding for learners who have difficulty. We'll examine how to use inclusion, so students can benefit from each other's insights, and we'll explore ways to use differentiation to provide *just right instruction* for everyone.

To begin a conversation about teaching math, we must first consider what math concepts are and how our minds learn them. If we are going to move from memorization to understanding, we need to know how the brain learns to make sense of and incorporate new mathematical ideas.

Some underlying ideas behind good math instruction

1. *Teaching math requires us to understand student thinking.*

Mathematics instruction has been a bit behind reading instruction in the sense that, when teaching reading, teachers are explicitly taught that they must understand learner strategies to inform instruction. Teachers need to know *how learners are making sense* of the math they do, rather than providing thinking for them to copy. By knowing how a student thinks about the concepts she is working on, we can help her find her way to new understanding.

2. *Students must "do the work" if they are to make sense of the math.*

There is a large gap between a student's personal understanding of a concept, however idiosyncratic, and a student's "borrowed understanding" from the teacher. We

2 AllLearnersNetwork.com

want teachers to understand that math instruction is not about students copying their thinking. It's about offering tasks and problems to develop understanding and then using questions and practice to help students clarify and extend their own insights.

This means that we must abandon the idea that we "model good math procedures" and then have students practice what we've done. In our approach, we take the opposite of the *gradual release* model of instruction: "I do, we do, you do." In teaching math for all learners, students explore a concept to make personal meaning and to gain conceptual insight. Sharing these insights with others refines and extends the understanding. Finally, the teacher might make connections between student understanding and the greater world of mathematics.

3. *What students have to say to each other is as important as what the teacher says to them.*

Student discourse is at the heart of learning. Human beings are social animals. Being part of a conversation that leads to understanding is motivating (even contagious!) for many learners. For some students, the process of interaction will enable a mathematical insight. For others, personal struggle leads to understanding, but that understanding is refined and extended through conversation with others. In either case, the opportunity for frequent student-student discourse is a touchstone of instruction aimed at serving all learners. Mathematical conversations also benefit students who are learning English, students who need more time to chew on ideas or students who – for any reason – might need support from others to make personal connections. Student conversations about mathematics in small groups is an important scaffold to support any learner who might need help with understanding and is an effective support for all learners - those who struggle and those we don't.

Learning Sequence

Besides the three cornerstones for instruction above, there is a trajectory that we use to describe the process of learning mathematics. While the considerations above are about pedagogy, the sequence below is about the learner.

1. *Mathematical thinking is based on conceptual models.*

Learners can approach mathematics either as a collection of procedures or as an approach to solving problems. A more traditional approach teaches students to use the same procedures (or *algorithms*) to solve problems. This can be summed up as "showing them how," and most of us were taught this way. An algorithm, by definition, is a procedure that requires no thinking. It's automatic. We program algorithms into computers so they can do the computation for us (more on that later). Taking an algorithmic approach to instruction means that we want students to recognize when to use the correct algorithm and then to apply it accurately. While there is still a place for developing algorithms and knowing when to apply them, real mathematics involves creativity and insight, which are key components of problem solving, logic, and pattern recognition: the real heart of mathematics.

Problem solving, or *heuristics*, is what math is really about. What good is knowing the quadratic formula, or the area of a circle, or how to multiply and divide fractions, if you never apply it to anything useful? Math is a human endeavor, as interesting and thought-provoking as literature or art. Teaching only algorithms is a bit like teaching a student to play scales on an instrument without allowing him to explore what it means to express himself in music.

These days there is a kind of algorithm/heuristic dichotomy, but as math teachers we *always* want students to work with understanding. How can we help someone to understand a concept without "showing them how"? We give them "tools to think with." These tools allow students to manipulate mathematical ideas, whether physically or mentally, to fit new situations or to solve new problems. We call these tools *conceptual models,* or just *models*. Models are the cornerstone of learning math. They allow us to think about abstract ideas, like numbers, using physical or represented images. Sometimes numbers and symbols themselves can serve as abstract models. The most basic example of using models as a tool to think with is the way that young children use their fingers to count all, to count back, or to add on.

A conceptual model is an analogue or prototype that behaves the way the math behaves. When I join 3 blocks together with 2 blocks, the total is 5 blocks. The blocks provide an analogy for the abstract act of reasoning additively. Likewise, the use of a double number line to explore proportions allows students to examine the concept of co-variation in a way that can be changed to fit a wide variety of situations.

Models come in three flavors: concrete, representational, abstract. Concrete models are those that can be manipulated physically. Representational models are those are drawn or recorded on paper (like number lines and T charts). Abstract models are those that make use of the manipulation of quantities symbolically. When students, for example, decompose numbers for computation (we'll learn much more about this later), they are manipulating abstract models: the symbols for numbers, in this case, carry the meaning. The numeral "3", for example stands in for the quantity of three. This makes the numeral a model for the number. Taking the number apart or redistributing it for computation without physical objects is an example of using abstract models.

Some educators believe that a concept can be more fully developed, particularly for learners who have difficulty with mathematical concepts, by following the concrete-representational-abstract models sequentially. That is, they suggest that there is a benefit to having students explore mathematical concepts in increasingly abstract ways. There is some empirical evidence to support this point of view. In our work on having students demonstrate understanding with all three models (see Tapper, 2012), we have found that students often prefer to use representational models when they are given the choice and are doing their own thinking. In any case, model use is essential for making personal meaning of mathematical concepts.

2. *Students use models to solve problems and, in doing so, develop mathematical strategies for solving new problems.*

A model is an effective tool, but a tool is meant to be used *for something*. In the case of models, we use them to solve mathematical problems. One of the biggest shortcomings in math education over the last 20 years is that teachers have sometimes taught the use of models without bringing them explicitly into problem solving. Learners can use a grouping model to solve problems about groups, usually related to multiplicative thinking. They can use an area model to figure out how to add or subtract rational numbers.

Posing tasks or problems is the heart of math instruction. Once a learner has a model to work with (with luck, *several* models in the toolbox), they need to apply this learning to a wide variety of situations and problems. When, for example, a learner can use unifix cubes to model a number sentence (5+3=8), they can then use the same model to find *all* the addends that will make a sum of 8. In this instance, the same model is being used to extend some understanding. The model is the tool, and the teacher's role is to facilitate student explorations of math concepts using that tool.

When students use a tool repeatedly in similar ways, they develop *strategies*. Decomposing into tens and ones (a common approach when using place value blocks) can become a strategy with consistent use. "Making 10's", "doubles, doubles plus one" are common strategies for adding that come from using models in specific contexts. For a concept to develop, then, a model has to be *used to solve problems*.

3. *Real understanding begins with a moment of conceptual insight.*

Learning math with understanding is not a linear process. The process is less like traveling along a straight path and more like wandering in a specific, if not predictable, direction. Problem solving creates productive struggle with conceptual models, which leads to realization. In the moment of realization, a learner is in the “flow” or having “aha” moment (Csikszentmihalyi, 2016). It usually comes from being stuck in a problem and engaging in productive struggle to figure it out. As teachers we are always looking for this moment of insight in our students. When students have a sudden insight, practice and conversation about the concept will cause their understanding to grow. When students attempt to practice before they have that moment of understanding, they can become progressively more tangled and invent less and less logical approaches.

One of the interesting things about *flow* is that it is connected to a particular state of mind, measurable with brain scans. Insight is far more likely when a person is in a relaxed state than when they are anxious. Students who struggle with understanding math frequently report heightened states of anxiety. Anxiety is likely one of the prime reasons for difficulty understanding, both from its effect on working memory and because it inhibits insight. Classrooms where productive struggle is routine, and where students expect to find problems challenging, produce far less anxiety. The anxiety in classrooms is sometimes a result of adult beliefs that answers must come quickly and easily. In mathematics, though, this is rarely the case.

4. *Opportunities to reflect on and communicate insights deepen understanding.*

When a student has had some insight into a new mathematical concept, there is benefit to refining and connecting that learning to prior knowledge and to a greater social context. This means that the math a student does has some usefulness, or meaning, in the real world. Connecting math to context is done through classroom discourse during the Main Lesson (see Chapter 2) and through deliberate practice during Menu (also in Chapter 2).

As students discuss and practice new understandings, they make connections to what they already know. “Percents are like a ratio”, “2/4 is the same as ½ because it covers the same amount of space on the same whole”, “Multiplying and dividing are both about making groups with the same number of things” are examples of the kind of connections that students can make through reflection or through conversation with other students. As students make these connections, they broaden their understanding of the newly-acquired concept.

Students can also learn to make connections to a mathematical convention or context. In his work on the Zone of Proximal Development, Vygotsky suggested that as we have new experiences, it's useful to have a teacher or another student help us to name our experience in a way that everyone can understand (Moll, 2014). For example, when I find that I've divided 27 things up among 5 groups and there are 2 left over, it's helpful to have someone tell me that those leftovers are called *remainders*. Learning this special language allows me to communicate my understanding with the greater body of math knowledge.

Many schools and educators embrace the maxim, "All Children Can Learn." In practice, not many behave as if this were true. Our approach in this book is to embrace the knowledge that when math instruction is skillful and positive intentions for student success guide our decisions (practice?), "All Children Can Learn" can become a reality.

The Big Ideas of math instruction in this book

- To teach math to all learners, we must allow them to develop their own thinking, rather than copying ours.
- We help students by introducing new concepts with models – tools to think with. We then have students solve problems using the models.
- Student understanding emerges when they solve problems and have mathematical insight. Insight means that they can "see" the concept from a more sophisticated perspective.
- After developing a mathematical insight, students benefit from reflecting on it, talking with peers about it, and practicing its application is a wide variety of settings.

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