CHAPTER 2

Anatomy of a Math Lesson

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An All Learners Lesson

There are lots of approaches to math lessons, and lots of ways to teach math. We propose a lesson structure targeted at both the learning of grade level mathematics and work with unfinished learning of prior concepts.

The goal of this lesson is to keep children working at grade level while also filling gaps from prior learning

The *All Learners Lesson,* is a structure that will serve the purpose of meeting diverse needs and increasing equity. People who use the *All Learners Lesson* in the classroom will often customize it in a way that meets their personal needs. Thousands of teachers have done this. But to begin, we'll look at the components of the *All Learners Lesson* and see how we can apply what we've learned to provide every learner with access to important mathematical ideas.

Inclusion and Differentiation

Math classes, like every other group of people who come together for a specific purpose, include learners with different strengths and abilities. The challenge (and benefit!) of this is how to meet everyone's needs. If you ever sat through a math class without understanding what was going on, you know how challenging this can be. If you've ever tried to teach someone some math, you can also understand that not everyone thinks the same way. What to do? Research on learning suggests that there are two seemingly opposite approaches that can help learners. These approaches are called, *differentiation*, and *inclusion*.

Most conscientious math teachers make a good faith effort to provide more advanced work for some students and easier work for students who are overwhelmed. By the time students are in high school this approach becomes tracking. "High" math students will take calculus. "Low" math students will take consumer math. Tracking is actually one of the most harmful ways that students are kept out of all the opportunities math can bring. Tracking is a failed idea, but the impulse that led to it – trying to give students work they can do effectively – is a good one. What's needed is a more refined approach that provides individualized, challenging, "just right" work for everyone. Finding what's "just right" for an individual student comes from understanding their thinking, one of the key components of effective instruction in any discipline. Differentiating math work is the result of understanding what a student (more often a group of students in the class) might need to move their understanding forward. On a day-to-day basis, differentiation allows teachers to meet learners' needs.

However, students cannot reach their full potential if they are only working on an individualized curriculum. Students learn better when they work together. They can

share their ideas, collaborate to develop new thinking, and challenge each other's points of view. Obviously, this is unlikely to occur if all students are working on their own differentiated curriculum. To do this effectively, students must have intentional and systematic opportunities to work in mixed ability groups. This is called *inclusion*. Studies on the value of inclusion suggest that there are benefits for everyone in heterogenous groups, both the students with deep understanding and those who struggle. When students talk to each other about their thinking, when they ask each other questions and debate approaches, everyone learns more.

The challenge then, is for us as teachers to provide both inclusion and differentiation or *just right instruction*[1] to our students. That's where the All Learners Lesson comes in.

Four Components of the All Learners Lesson

The lesson we use in *Teaching Math for All Learners* has four parts, two shorter and two longer. These include:

- Launch
- Main Lesson
- Menu
- Closure

The purpose of this chapter is to give you a clear idea of what each of these components looks like, how it functions in the lesson, and how it is connected to the important ideas pulled from math learning research. When you are finished reading, you should have a better sense of how to structure a lesson to meet the needs of all learners.

The Launch

Math lessons start by activating prior knowledge. This means getting students ready to approach a new concept by helping them recall and organize concepts and skills they already know.

Think of a time when you were learning something new – how to drive a car, ride a bike, connect a Bluetooth device, or swim. When you've just learned the

1 The term, *differentiation*, can have lots of meanings. For example, providing an open-ended lesson that is accessible to everyone is a kind of differentiation. In our definition for this book, we mean providing work at a particular student's level of proficiency. In reading this is sometimes called, "just right" work.

beginning of a new skill you tend to remind yourself of the things you'll need to remember.

> *OK. I've put on my seatbelt. I've adjusted the mirrors. I've checked for cars and people around me. Now I can put the car in gear and pull out…*

Launching a math lesson is similar in many ways. It's a quick way to get students to remember what they already know (activating prior knowledge) and it's a way to introduce the more substantial work of the main lesson. It usually takes about 5-10 minutes. Taking too long to launch a lesson is a common mistake that many newer teachers make. It's good to remind ourselves that the launch gets us started, but the heavy lifting comes right after.

A good launch will usually fall into two categories: a small problem or a number talk. Despite the short duration of the launch, both of these approaches are surprisingly complex. Younger students, for example, might engage in small problems that involve counting and/or patterns with the calendar. A launch for sixth graders sometimes involves a "do now" problem that students complete independently and share. In both cases, the goal is to get the learner's mind focused on the ideas that will be explored in the main lesson.

What happens during a Launch:

- EVERYONE should have some way into the problem
- There should be an opportunity for sharing. The emphasis should be on *strategies* rather than solely on answers. The more strategies that come out during a launch, the more successful it is.
- The launch should activate prior knowledge.
- The launch should introduce important concepts central to the main lesson.

Launches use a small problem or a Number Talk to activate prior knowledge and introduce the Main Lesson.

<u>Using a small problem for the Launch</u>

Let's look at a launch for a typical third grade class that uses a small problem.

> *How many legs do 4 spiders have?*

What does a teacher do when launching a lesson with a small problem?

The teacher would give the students 1 minute or so to think about this for themselves. She might, then, ask students to do a "turn and talk"[2] with a partner to share what they've thought about the problem. Then she would ask students to share their thinking.

Spiders have 8 legs, so I drew 4 circles with 8 legs each.

I did 8+8=16 twice. That was 32.

I know that 8 is two less than 10.

So I did 40 and took away 8 because that is 4 twos.

The teacher will ask students to share all the strategies they can and, when appropriate, she will record some thinking on the board. In addition, she will encourage discourse (student-student talk) by asking students to explain each other's thinking (revoicing).

Can anyone explain Carly's strategy a different way?

Or challenging students to reflect:

Can anyone tell about a strategy that was new to them? How is it different from what <u>you</u> did?

When creating a launch, small problems that encourage multiple strategies help students get ready to solve more complex problems and learn new concepts.

<u>Using a Number Talk for a Launch</u>

A Number Talk is similar to a small problem but makes use of computation and mental math to activate prior knowledge and encourage the use of multiple strategies. A great deal has been written about Number Talks (Parrish, 2014) so this is a topic we will cover only in basic detail. There is no paper or pencil used for this work. All the computation is done mentally with the teacher recording student strategies on the board.

What does a teacher do when launching a lesson using a Number Talk?

1. Write a numerical expression on the board. (12x25 – for example)

2 Turn and talk is a simple "talk move" where students discuss a concept with a student (or students) sitting near them. Sometimes this is called, "pair share" or something equivalent. It's short and meant to help introduce ideas for a larger group conversation.

2. Ask students to put a thumb in front of their hearts when they can think of one way to solve the expression. They should raise a finger when they can think of a second way.
3. Let students think about how to solve until at least two thirds of the class has their thumbs up.
4. Ask for answers that students have for the expression. Write each of these in a corner of the board without comment.
5. Accept ***ALL*** answers.
6. When everyone in the class is happy with one of the answers on the board, ask for volunteers to "defend" one of the answers.
7. Record student thinking as students share their strategies.

Number Talks demand that the teacher

- Exercise good wait time. (Give students enough time for them to think of how to solve the expression)
- Except all answers without commenting on their correctness (This is taken care of through conversation about strategies)
- Encourage multiple strategies. (There is always more than one way to solve an expression mentally. The whole class benefits from brainstorming multiple strategies)

Number Talks are one of the most effective ways to launch a lesson. They encourage students to think strategically about computation and they allow them the chance to see how other students solve. Getting a chance to watch the strategies of others leads directly to new learning and can often increase the efficiency of strategies students already know.

Why are strategies so important?

In all the elements of the math lesson we present here, there is an emphasis on students developing, and sharing, their strategies for solving. Among good math teachers this is a common practice, but it is still relatively uncommon in the world at large. In general, elementary teachers seem to feel that their job is to present math concepts and procedures as clearly as they can, and then have students practice the methods they've been shown until they can demonstrate mastery. At first glance, this seems like a logical, even obvious, approach to instruction. The argument goes, "Show students the best way to solve particular kinds of problems and then have them practice doing it." So why not teach this way?

You don't have to talk with too many people about their understanding of math to see where this approach falls down. In conversations with adults, one often hears the phrase "I don't remember the formula for that." Or "I don't remember how to do that." While understandable, most math is about understanding ideas – not remembering formulas. American students persistently demonstrate that they are capable of (rather slavishly) applying formulas when they've been shown what to do. But they have difficulty solving problems when they don't know what formula to use ahead of time (Stigler,1999; Wolper, 2018). For many students, math is a trick they can do when asked to. It is not a broad understanding of concepts – a particular lens through which to see the world. As a result, students can't really use math, they can only "do it" when they're in school.

An emphasis on helping students develop strategies that make sense (to them) ensures that math is usable and accessible. They are not "borrowing" the teacher's thinking. Rather, they are using what they already know to make new meaning of a new problem or expression. The guiding motto for this approach to instruction is:

"How would *you* do this?" Rather than "This is how you do this."

Not only does this approach help students make better cognitive connections between concepts, which leads to better math performance in both computation and problem solving (Geary, Hoard, Nugent & Byrd-Craven, 2007; Vicich, Knott & Evitts, 2007) but it reduces the anxiety that students have about learning math because they are almost never asked to replicate something they don't understand.

Number Talks help students better understand computation for a number of reasons.
A Number Talk asks students to think of their most efficient strategies because, as all Number Talks involve mental math, they must hold all the information for solving in their heads. This can be a problem, though, for some students who struggle with math.

Students with working memory challenges, for example, are limited in their ability to work with lots of information at once. Further, when students are anxious, working memory resources tend to shrink. Since Number Talks require the use of working memory and can make students anxious if they're not sure what to do, it can be difficult for some students to participate in them. One powerful solution to these issues is to pre-teach, that is, to let students work on material individually or in a small group before they encounter it with the whole class.

We will learn later in this chapter about a portion of the lesson called Menu, where students receive differentiated, or individualized, instruction. This is a time when the classroom teacher, a learning specialist, or a paraprofessional can take time and work with students on the following day's Number Talk. Working individually, or in a small group, students can take more time to consider how they might solve the expression in the next day's Number Talk. If necessary, they can even use drawings or concrete materials to help with understanding. The next day, when the rest of the class takes on the Number Talk, these students will be prepared to contribute strategies with confidence. This helps with both understanding the math shared by other students, and with the confidence and competence needed to be successful.

Main Lesson

What happens during Main Lesson?

Main Lesson is sometimes referred to as a "Mini-Lesson". It is the heart of new learning, a time for students to encounter new ideas and (in small ways) transform their thinking. It takes up about 20 minutes of the full lesson.

During the Main Lesson, a grade-level problem or task (see below) is presented to the whole group. After introducing a problem to the group, the class is divided into small *heterogeneous* groups. These groups work together to come up with a strategy

and solution. These strategies are then shared with the whole group, usually through students putting their work on the board, and discussed.

Main Lesson is the *inclusion* element of the lesson. It makes use of problem solving and student discourse to develop conceptual understanding of mathematics. Main lesson is the time for new insights, for making connections between previous understandings and an expanded view. It's not a time usually used for practicing or review. This is important, as one criticism of American math curriculum and instruction is that a good deal of time each year is spent reviewing previously learned concepts. Countries with higher math performance spend much more time on new learning (Stigler, 1999).

What is inclusion and why is it important?

Children learn far more in school than just the subjects we teach them. They learn to see themselves as part of a larger group or community. When students are working on mathematics, they often develop self-images that don't support learning. Some students *learn* to believe that "math is hard", or "I'm not smart enough to do this," or even, "I just want to get out of here." These feelings toward math (which are very common) don't come to school with children. They are developed there. Some of the practices that can create a negative relationship with math include:

- Tracking students into long term ability groups
- Having students compete against each other (timed tests, wall charts with rankings, etc.)
- Showing that there is (only) one correct way to solve (or think about) a math problem Even telling students they're "wrong" – without helping them to think through their ideas

Inclusion can be an antidote to students developing anxiety about math learning. The idea is that children will get a different message if they're allow to work together, share ideas, collaborate on strategies, and solve *together*.

Main Lesson is always focused on *grade level math*. At this point in time that means that the work that students do covers the complete Common Core State Standards . Main Lesson is when teachers make sure that what the math students get (from a program or teacher-created) is *comprehensive*. In the best of all worlds that would mean that every student would have good understanding of all those concepts. In

reality, however, not all the concepts in a comprehensive curriculum are equal. We'll discuss more on this when we look at High Leverage Concepts.

For all learners to be exposed to, and work with, grade level mathematics is important. In instances where well-meaning schools or programs put students into remedial programs where they work only at math at their level, students are always behind. For this reason, there needs to be a time for working on grade level math (Main Lesson) and working on just right instruction (Menu).

The key elements to inclusion in an All Learners math lesson are:

1. Everyone is working on the same math.
2. The problem or task chosen is accessible (a "low floor, high ceiling" type of problem)
3. Student to student discourse is *taught*, to encourage high quality discussions
4. Problem solving is collaborative.
5. Groups share their strategies with the class.

Main Lesson

What does Main Lesson look like?

1. *Everyone is working on the same math.*

Since this is the inclusion part of the lesson, the task or problem is the same for everyone. The scaffolding, or support in the lesson comes from the way the students work together. Since everyone must do the same lesson, it's important that the lesson has a way for everyone to attempt it.

2. *The problem or task chosen is accessible* (a "low floor, high ceiling" type of problem).

The way teachers can make Main Lesson work accessible is by using problems that have "low floors" and "high ceilings". These are problems that have many ways for learners to begin solving and allow the use of many strategies. Three-Act Math problems[3] are a good example of problems or tasks that allow students to explore a math concept in a wide variety of ways.

3. *Student to student discourse is **taught**, to encourage high quality discussions.*

3https://gfletchy.com/3-act-lessons/

It would be great if students came to school knowing how to collaborate and problem-solve together. Most of the time, though, they need to learn how. When teachers support all learners, they help to facilitate student-student conversation during problem solving. Sometimes this means that teachers will design procedures or protocols for students to use the help get the conversation started. An example of this is when teachers ask each person in a group to say what they notice about a problem before the group begins to work on it. Teachers pay close attention to whether each group member has to opportunity and ability to contribute to the group conversation. If he/she doesn't, then the teacher finds ways to help.

4. *Problem solving is collaborative.*

There are students who have deeper insight into math. They need a chance to develop that – but Main Lesson is not always the place to do that. (Menu is better) The problem solving is about what the *group* can do. Each member of the group is expected to be able to explain/demonstrate the group's solution. No one should get left behind. While it's true that some students will have a greater role in some problems than others. The overwhelming sense of the work is that everyone is helping.

5. *Groups share their strategies with the class.*

Since every group will have some kind of strategy or approach, these should be shared with the whole class at some point in the lesson so students can see the thinking of their peers. One of the most powerful tools a teacher has is to ask students to try to understand someone else's thinking. Comparing strategies used on the same problem is one of the most important ways that students learn to expand their understandings. As they encounter other strategies, they will often learn to be more efficient and more abstract. This comes from seeing other (sometimes more advanced) work and from thinking more deeply about the concept being learned by seeing it from several different perspectives.

What does a teacher do during the Main Lesson?

The Main Lesson is much like what most people think of as "math class", with a few important exceptions. The class is working as a whole group. The teacher leads the class in a problem solving experience. Students work together and ask questions. When teachers are just learning how to teach a math lesson, Main Lesson can look like

teaching a lesson from the school's math program. While teaching for all learners can look very similar to this kind of lesson, it varies from this in some important ways.

Since a Main Lesson is always about new conceptual learning with a problem or task, the first thing a teacher does is to introduce the problem. This is done in with a specific technique called the Problem Introduction Protocol.

PROBLEM INTRODUCTION PROTOCOL

- Read the problem chorally.
- Ask, "What are we trying to figure out?"
- Ask, "What would an answer to that look like?"
- Brainstorm strategies.

Read the Problem Chorally

We do this to accommodate students who might have difficulty with reading and those whose first language is not English. Older kids fuss a bit about reading chorally, but it's important they do it anyway. It's important that students get a chance to hear a clear reading of the problem.

Ask, "What are we trying to figure out?"

Step 2 involves determining what kind of answer we're looking for. Many teachers write a statement on the board to summarize student thinking. Some teachers have students write a statement on the paper where they're going to solve. Some (teachers of younger children) will write it on the board and ask students to copy it onto their papers.

Whether it's written or not, students should all be able to articulate what the goal of the problem or task is.

Ask, "What would an answer to that look like?"

In this step, we ask about two elements of the problem:

What units will the answer have?

What's a ballpark estimate for the answer?

The estimate is usually the result of a teacher question about extreme (and unreasonable) answers. Teachers will ask questions like, "Could it be 1? Could it be 100?" By eliminating unreasonable answers we're helping kids narrow their thinking a bit.

When this step is done, students should know what the units for the answer will be and what a reasonable solution might look like.

Brainstorm Strategies.

Students now suggest strategies for solving the problem or attacking the mathematical task from the Main Lesson (addition, making a list, drawing a picture, multiplication, etc.) It's important to respond to these suggestions by saying, "We might be able to use that strategy." We don't want to discourage any particular approach or push students toward one particular way of solving. "That won't work," is an example of the former and, "Yes, this is a subtraction problem," is an example of the latter. We want students to feel comfortable suggesting any strategy that seems reasonable *to them*. This is a way of validating attempts at solving. We also want to steer clear of telling students there is one way of doing any problem. Any problem that can be solved through subtraction, for example, can be solved using addition, a T-Chart, direct modeling, or a picture. It's a good idea to record the strategies students suggest on the board. As students are getting started, they can refer to these.

Now that students have read, understood, and developed strategies for solving, the teacher's job is help students talk to each other (in their groups) about ways to solve. There are other resources that explore student discourse in greater depth, but here are some basics to promote good conversations between students:

Get everyone to say something about the problem before starting to solve.

Problem solving is much less about creating plans to solve and much more about understanding the problem. The more insight a group can develop about a problem, the more insight they will develop and the easier it will be to solve.

To start the process of deeply understanding, have each member of the group say something they notice about the problem. This doesn't need to be a way to solve. It

can simply be a way to better understand the problem. Some things that students might say at this point include:

This looks like comparing how fast each of these cars go.

I think we have to find the area before we know how much carpet we need.

We won't know if there is enough candy until we figure out how much everyone needs.

There are groups in this problem.

Isn't this, "miles per gallon"? Didn't we do a problem like that already?

After making observations, students can begin to solve.

Sometimes, in an effort to better understand the problem, a teacher will work with a group to suggest other mathematical questions that could be asked using the same situation as the problem they're trying to solve. For example, if a problem reads:

Tannie is having a birthday party. She wants to buy yo-yos as party favors. If there are 6 guests coming to her party, and yo-yos are $2.45, how much will Tannie spend?

A teacher might ask, what are math questions could we ask about this problem?

What if the yo-yos are $10?

What if there are 2 guests instead of 6?

What if she wants something different, like pencil cases or gift cards?

Exploring other situations related to the original context can sometimes help children understand the problem more deeply and give them insight into solving.

Finally, if a group is really stuck, the teacher can use the Problem Introduction Protocol again with the group to see if they can find a strategy they can all work with. The rule for solving as a group during Main Lesson is that *everyone* should be able to explain the answer the group found.

When groups have found a solution, it's time to share work. Sometimes this is done in the same lesson as the introduction of the problem. Sometimes it happens the next day. There are several ways that students can do this. The easiest way is to have

one member of the group come to the board and put the group's solution. If there are four groups working in the classroom, then all four representatives will come to the board and put their work up at the same time. They will talk about their strategies with the whole group and answer questions.

Technology affords other ways of students sharing their work, though. A document camera can be used to project work from a student notebook onto a screen. Similarly, teachers sometimes take a picture of student work with their phones or tablets and project the work that way. Finally, if students work from a tablet or computer, a Wi-Fi or Bluetooth connection makes it easy for students to share their work with the class.

When debriefing problem solving there are ways that the classroom teacher can help encourage students listening to each other. We'll get more deeply into this in the chapter on student discourse, but one simple move that teachers can use is called, "revoicing." Revoicing is when a teacher asks a student from another group to talk about work that is not her own. Asking students to defend the logic in someone else's work is a good way to get them to tune into what their peers are doing. It can also help clarifying the work as new eyes can help explain elements of the strategy that may not come out if the original authors talk about it.

When strategy sharing has taken place, it's time for students to move toward more individualized instruction. It's time to move into Menu.

Menu

For math instruction to really be for all learners, it must meet individual needs effectively. A completely individual program, though, can limit the amount of time that a student has for the important interaction that comes from group with students of different abilities. The All Learners lesson is our attempt at balancing *just right* instruction with productive conversations.

Menu has two critical components: student choice, and work that is differentiated, or geared toward individual students. Student choice is important because it can be motivating and because it changes the notion of who the math work is for. So often children perform mathematical tasks in the classroom because they're told to do so, or because they're threatened with consequences if they don't. There are

reasons why teachers want students to learn math, just like parents want their children to eat kale: both are good for children in the long run. The problem with motivation that relies on, "Because that's what's best for you," is that when parents are no longer present, students will do what they really want to do.

Perhaps a better approach to motivation is to offer choices. When a parent says, "You can have kale, or broccoli, or spinach, she is not leaving open the possibility of not eating greens. But she is giving her child the chance to take some ownership by making her own decision. This is true for math, too. If children are offered opportunities to make choices about how they do their math work, they tend to take more ownership of it. If the work is interesting (to them), they may even be more inclined to engage with the work, to make personal meaningful from it. Many teachers who use Math Menu report that their students love it.

While motivation is an important element to Math Menu, the instructional intention is for students to work on math that is right on their level. This is work in the Goldilocks zone: Not too hard, not too easy---just right. Menu is a time when students work on review, get help for areas of difficulty, and/or explore areas of math that are important for them individually. For students who are a bit behind, Menu is a time for getting help to catch up with grade level expectations. For students who need more challenge, Menu is a time when they can dive into problems that require deep thinking and mathematical skill.

What does Menu look like in a classroom?

Because of the way it's managed, Menu can look quite different from one classroom to another. Teachers organize student choice in a wide variety of ways depending on the students, their own preferences for management, and their goals. In general, though, when you walk into a classroom doing Math Menu you see a group of children engaged in a wide variety of activities. Some of them are working in groups, some are working alone, and some are working with a teacher or other classroom instructor. There is a sense of business and purpose. The teacher will work with an individual student or a small group for a few minutes and then spend some time walking around the room, answering questions, and helping to keep students focused on their mathematical tasks. For the teacher, there is a movement from small group instruction to keeping an eye on the class as a whole.

For those beginning to use Menu tasks are often designated as *Must-dos* and *Can-dos* to delineate between required and optional activities. The required activities sometimes vary from student to student, though the bulk of the class will be required to complete the same assignments. Students on IEPs or those who need to be challenged might have a Menu that is distinctly different from others in the room. At times, these adjustments are small – like reducing the requirements or adding an additional task. In some cases, though, a student might have a completely different Menu. These are instances in which the student's needs are quite different from his peers. Some of the most important successes that have come from Menu have taken place when a student who previously received instruction in a separate environment can join her peers and do work at her own level.

Some Suggestions for Required Activities

The suggestions for Menu listed here are activities that emphasize important elements of math learning. They are not meant to be an exhaustive list, but rather a starting place. Most Math Menus will have far more than these four activities. The required activities we recommend include:

- Problem solving
- Skills Practice
- Games
- Math Journal

In all these activities separately, or as whole, the principle components of Menu are applied. There is choice and there is differentiation. Choice can be structured in different ways that reflect how individual teachers organize their classrooms. The choice can be applied to the Menu as a whole – students can decide whether they work on a problem or game, for example, during their Menu time. In this approach, teachers turn children loose to make their own decisions, as long as they complete the Menu.

Another approach is to have students move in groups. Some teachers, for example, have students move from "station to station". In this case, choice can also be applied to a station in each of these four areas – a student may get to choose among several appropriate problems or from a variety of games. At the problem-solving station students will have choices about the problems they solve at their own level.

Similarly, students who are at the Math Journal station could find a variety of choices of writing prompts to reflect on.

Differentiation takes place by offering *just right* work to students to help them review prior learning, practice new concepts, or get extra attention for topics they've struggled with. In its simplest form, differentiation can be present in a Menu by simply offering different levels of the same work. For example, a teacher might offer the same word problem at the problem-solving station with versions that are more accessible or more challenging. This is usually accomplished by changing the numbers in a problem (an additive problem with 2-digit numbers as opposed to 3-digit numbers) or adding steps for solving (to make it more challenging).

A more effective approach that teachers take, when they have more experience with Menu, is to create specific activities based on formative assessment. We'll visit this in greater detail in Chapter 4. For now, we should understand that the ideal for Menu is that, in addition to some practice, reflection, and review we might offer students, there could be at least one activity that is geared specifically to their instructional needs. The first step in the evolution of Menu is offer variety. The second is to include choices that address individual needs.

Problem Solving

Problem solving is the single most important activity students do in math. It helps create understanding, provides reflection, and allows for concepts to be generalized to new situations. Problem solving during Menu is a little different that in the Main Lesson. Often, though not always, students will be solving problems during the menu independently or with a friend. For this reason, problems in Menu must be more accessible than those in the Main Lesson. They should be aimed at the level where students can work on their own or with the support of a peer. The exception to this is when problem solving is part of small group instruction led by the teacher.

An important element of problem solving during Menu is the idea of *interleaving* . This means that not all the problem solving in Menu should be on the same concept as that being studied during the Main Lesson. For example, if the Main Lessons have been focused on multiplication, some of the problem solving in Menu might involve additive problems, like comparisons. The goal is to keep students from

pulling numbers from a problem and applying the mathematics *du jour*. Students who struggle often have the hardest time deciding which operation to use when solving a word problem. Interleaving helps students by making them ask which strategies make the most sense for a given problem.

Problem solving, from an instructional point of view, is about strategies (more, even, than answers). The most important goal of problem solving is for students to understand the problem enough to be able to approach solving in a way that is meaningful *to them*. The development of strategies is the development of logical thinking and, perhaps, the best way for students to learn to think in more complex ways.

Problems for Menu can come from a wide variety of sources. The internet is full of resources for problem solving. Not all are wonderful, but most are at least a starting place for good problems. Most often, teachers find problems in the math program they use that can be adapted to Menu.

Problems that students solve are the most common artifacts that teachers keep from Menu. Other practice assignments are corrected and sent home. But problem solving gives a good deal of insight into the way a student is thinking mathematically. Teachers often keep problem solving in a folder or portfolio as a record of a student's mathematical development. These samples can be revealing when reviewing a student's work for intervention, or for sharing with parents at parent-teacher conferences.

Skills Practice

Skills generally refers to arithmetic. This is one area where there are many, many resources available to teachers. There are digital resources that can be used (both paid and unpaid) and math programs frequently provide this level of practice for students. There are some important caveats to providing math practice for students. Timed practice can cause more problems than it's worth. And practice needs to be specific to the most important goals at each student's ability level.

Timed tests, once thought a useful way to gauge automaticity, create problems for many students. There is nothing about speed that's truly important in mathematics. In fact, an emphasis on speed is an emphasis on automaticity. While having automatic recall of facts is desirable, most students benefit from have a deep understanding of

how math facts are constructed and "go together". Emphasizing speed over depth thwarts efforts at understanding in favor of memorization. This is almost always a bad idea for instruction. Understanding should always be the most important goal.

Skills work should focus on the most important concepts at each grade level. In the All Learners Network, we believe that the most important concepts at each grade level are the ones that will help a student be successful at the next grade level. We have identified these as *High Leverage Concepts*. Chapter 3 gives a longer explanation of them. When we are working on skills, we use the HLCs for the grade level most appropriate for a student to guide decisions about what he should work. If a student can demonstrate understanding of key elements of an HLC at his own grade level, we provide practice in those skills practice for them. If they have some skills that are not solid from a previous grade, we might mix in some practice from previous HLCs to help the student practice skills that are most appropriate.

Just like with problem solving, the principle of *interleaving* should help inform decisions about work to give students. If Main Lessons are on division, some practice on multiplication and subtraction could be mixed in to be sure that those skills remain accessible.

In the All Learners Network the majority of skills work is provided through online platforms like Dreambox ™, iXL™, or River Deep ™, etc. These platforms work well in a Menu setting because they differentiate practice for students automatically. They also record the results of student work for teachers. In cases where these programs are in place, the Skills portion of Menu is just a matter of assigning students screen time to make use of them.

Games

In years past teachers created many homemade games for their students. Some programs, like Everyday Math, include games in their programs. This is another area of instruction where online resources make planning for Menu much easier than in the past. Web sites abound with games that children can play to practice some math. These online games can roughly be categorized as, "practice games" and "games that offer more."

Practice games are more like Skills practice. For that reason, some teachers use them in the Skills part of the lesson. These games are characterized by students having to solve various types of challenges by answering math fact or arithmetic problems. These types of games are useful but are often not as valuable as more complex games.

Games that offer a little more are games that involve application of mathematical concepts or the use of logic or strategy. In these games the student is challenged to think and make plans, rather than simply remember procedures or facts. Games that offer more can be useful to teachers for individual or small group instruction, as the teacher can ask the student why she made the strategic decisions she chose to make, and/or if other decisions might have been more successful.

While there are paid sites that offer complex game, some of the best resources are free. The National Council of Mathematics offers a variety of complex mathematical games for both computers and tablets without cost. These can be found in *NCTM Illuminations*[4]. NCTM also offers an interactive game site, *Calculation Nation*[5] that allows students in one classroom to play with other math learners around the country.

Math Journals

Reflection on mathematical understanding is a critical tool for connecting new insights with prior understanding. A moment of mathematical understanding can be interesting but, if students don't have the chance to make connections to other knowledge, their experience, and the understanding that came with it, can fade. The best way to support robust understanding is to give students a change to practice and reflection on new concepts they've learned. Reflection can be done in conversation with other students or through writing. Math Journals provide a venue for students to be metacognitive, by reflecting on their own understanding.

Writing in a Math Journal is scaffold using a prompt. The prompt is a short question or statement that asks students to tell what they think.

Some people say that addition and subtraction are opposites. Some people say they are really the same thing. What do you think? Why?

4https://illuminations.nctm.org/Games-Puzzles.aspx

5https://calculationnation.nctm.org/Login.aspx?ReturnUrl=%2fGames%2fdefault.aspx

Alyssa says that whenever you multiply a number by an even number the answer is always even. Is this true? Why do you think so?

Maria says 4 + 3 is the same as 3 + 4. Is she right? Why do you think so?

These are all examples of journal prompts that give students a chance to reflect on their understanding of important mathematical concepts. The prompts are almost never simply another math problem or question. Rather, a good prompt asks students to consider something they already know in order to go a little deeper with it.

Math Journals can also be a rich source of formative assessment. What students have to say about important math concepts tells teachers how they're thinking about big math ideas. Journals can provide teachers with an opportunity to offer more specific Menu work to the children in their classes since what students have to say points directly to their understanding.

Closure

Closure is primarily a good conversation about recent math learning. It's the place in the lesson where students have a chance to share their thinking, successes, and questions with each other. Like Math Journals, closure provides a chance for reflection on important ideas. It's a kind of "out loud thinking" that let's everyone share what they've learned and, potentially, influence the thinking of their peers.

What does Closure look like? What does the teacher do?

Closure happens with the whole class. Students are gathered together (often on the rug when they're younger) and the teacher facilitates a conversation where students share their experiences, their conjectures and their questions. Sometimes the teacher will record these. Often, she listens and finds ways to get the students to respond to each other. There are several important instructional moves teachers can use for closure:

- Sharing
- Responding to an observation
- An exit slip

Sharing

By far the most common approach to closure is to have students share their experiences in the larger group. An easy way to structure this sharing is to rely on two tried and true questions:

Why did you notice (while you were working)?

What did you wonder? (What questions do you have?)

During sharing, students share their experiences and the new questions they have as a result of the work they've done during the Main Lesson or the Menu. Often this will take the form of what individual students (or small groups) "found out" while they were working.

Student: I was doing my addition problems with Honor and we found out that, if you want to add 9 you just have to add 10 and take 1 away.

Teacher: Wow. (To the class) is that true? (Some thinking and then hesitant nods). Why would that be true?

Sometimes children just want to share that a problem was "really hard" or "I figured it out!" This time is important for developing a culture of sharing and celebration. It's also a great way for a teacher to probe individual conjectures to get the children thinking about bigger ideas.

Responding to an Observation

Another way that teachers can facilitate good conversations during closure is to report observations to the class for comment. Some teachers keep a notebook (or post-its) handy as they move through the room during Menu. When a student says something interesting, they record it in the notebook to use for closure. Here are some examples:

George says that the Shower Problem is weird because as you make the shower value slower, the length of the shower goes up. He says, "the numbers change in opposite directions." Is this true? What does he mean?

Silvia says that multiplying fractions is confusing because it makes the answers get smaller. Is she right? Why would this be?

Antonio says that even numbers means you can share easily. Is that true?

Responding to observations is very similar to sharing in that students respond to conjectures that emerge during the lesson. In this approach to closure, though, the teacher guides the discussion a bit more because she is not waiting for one of the students to make a conjecture to get the conversation going. She has collected good conversation starters for them.

Using an Exit Slip

Exit slips are short problems or questions that ask students to demonstrate their current understanding of the math concept they've just explored. An exit slip should allow students to show what they know in a relatively short period of time – about five minutes. Teachers will collect these and sort them by characteristics of the work, often using this information to plan Menu activities for the specific needs of each student.

Since exit slips require more teacher time than other methods (teachers will read the slips and use the information to plan instruction) they are often used less frequently than the two kinds of closure described above.

A final work about Closure…

Closure is a part of the lesson that's sometimes challenging to get to. When a Menu has engaged children and they're working productively, its difficult to stop and get them to collect themselves and have a good conversation. For this reason, teachers in the All Learners Network find that Closure doesn't need to happen every day. They find that sometimes the best time to have Closure is to use it as a Launch for the next day's lesson. Using the techniques described here, they will begin the new lesson by asking students to discuss what they learned the previous day.

Teachers in the ALN also felt that productive, longer conversations are necessary when coming to the end of a unit of study. For example, when the class has been working with multiplicative reasoning for two or three weeks, it's felt that time should be set aside to have a more thorough discussion of the important insights students have gained during the unit.

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